Basic Mathematics(Pre-requisite)

Differentiation

Differentiation is the process of obtaining the derived function f'(x) from the function f(x), where f'(x) is the derivative of f at x.

The derivatives of certain common functions are given in the Table of derivatives,
Table of derivatives :

$f(x)$	$f'(x)$
x^n	$nx^{n−-1}$
sin x	cos x
cos x	−-sin x
tan x	sec^2 x
cot x	−-cosec^2 x
sec x	sec x tan x
cosec x	−-(cosec x)(cot x)
ln x	1x1x
e^x	e^x

Many other functions can be differentiated using the following rules of differentiation:

(i) If $h(x) = k\,f(x)$ for all x, where k is a constant, then $h'(x) = k\,f'(x)$.

(ii) If h(x) = $f(x) + g(x)$ for all x, then h' $(x) = f'(x) + g'(x)$.

(iii) The product rule: If $h(x) = f(x)g(x)$ for all x, then h'(x) = $f(x)g'$(x) + $f'(x)$+$g'(x)$.

(iv) The reciprocal rule: If $h(x) = 1/f(x)$ and $f(x) \neq 0$ for all x, thenh'(x) = −f'(x)(f(x))2h'(x) = -f'(x)(f(x))2

(v) The quotient rule: If $h(x) = f(x)/g(x)$ and $g(x) \neq 0$ for all x, then
h'(x) = g(x)f'(x)−f(x)g'(x)(g(x))2h'(x) = g(x)f'(x)-f(x)g'(x)(g(x))2

(vi) The chain rule: If $h(x) = (f \text{ o } g)(x) = f(g(x))$ for all x, then $h'(x) = f'(g(x))g'(x)$.

Integration

Integration is the process of finding an anti-derivative of a given function f. 'Integrate f' means 'find an anti-derivative of f'. Such an anti-derivative may be called an indefinite integral of f and be denoted by ∫f(x)dx∫f(x)dx.

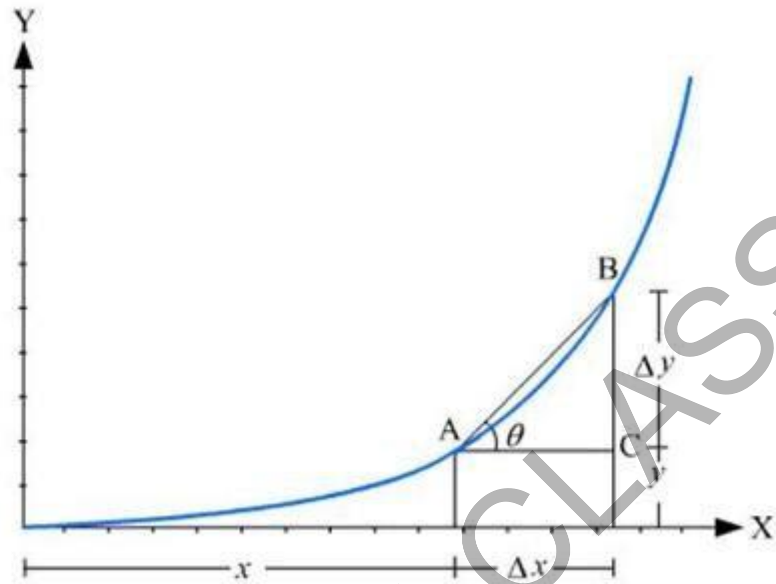
The term 'integration' is also used for any method of evaluating a definite integral.

∫baf(x)dx∫abf(x)dx

The definite integral can be evaluated if an anti-derivative Φ of *f* can be found, because then its value is Φ(*b*) − Φ(*a*). (This is provided that *a* and *b* both belong to an interval in which *f* is continuous.)

However, for many functions *f*, there is no anti-derivative expressible in terms of elementary functions, and other methods for evaluating the definite integral have to be sought, one such being so-called numerical integration.

Differential calculus



Let *x* and *y* be two quantities interrelated in such a way that for each value of *x* there is one and only one value of *y*.

The graph represents the *y* versus *x* curve. Any point in the graph gives an unique values of *x* and *y*. Let us consider the point A on the graph. We shall increase *x* by a small amount Δ*x*, and the corresponding change in *y* be Δ*y*.

Thus, when *x* change by Δ*x*, *y* change by Δ*y* and the rate of change of *y* with respect to *x* is equal to $\frac{\Delta y}{\Delta x}$

In the triangle ABC, coordinate of A is (x, y); coordinate of B is ($x + \Delta x$, $y + \Delta y$)

The rate $\frac{\Delta y}{\Delta x}$ can be written as,

$$\frac{\Delta y}{\Delta x} = \frac{BC}{AC} = \tan\theta = \text{slope of the line AB}$$

But this cannot be the precise definition of the rate because the rate also varies between the point A and B. So, we must take very small change in *x*. That is Δ*x* is nearly equal to zero.

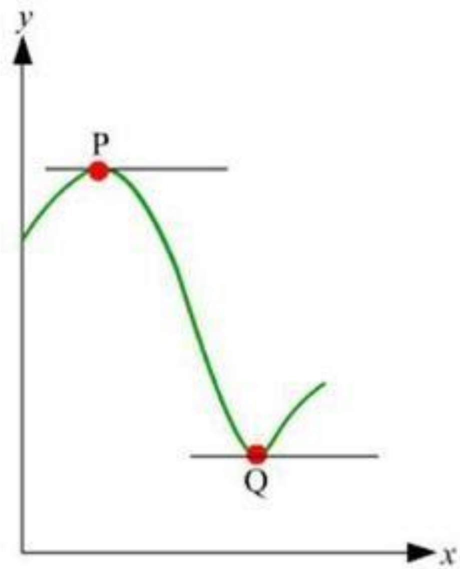
As we make Δx smaller and smaller the slope tanθtanθ of the line AB approaches the slope of the tangent at A. This slope of the tangent at A gives the rate of change of *y* with respect to *x* at A.

This rate is denoted by $\frac{dy}{dx}$ and,

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Maxima and Minima

Let *x* and *y* be two quantities interrelated in manner as shown in the graph below:



At the points P and Q the tangents to the curve is parallel to the x-axis.
Hence, its slope tanθtanθ = 0.

But we know that the slope of the curve at any point equals the rate of change $\frac{dy}{dx}$ at the point.
Thus, at maximum (at P) or at minimum (at Q),

$$\frac{dy}{dx} = 0$$

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. This implies, $\frac{dy}{dx}$ decrease at a maximum.

$$\therefore \frac{d}{dx}\left(\frac{dy}{dx}\right) < 0 \text{ at maximum}$$

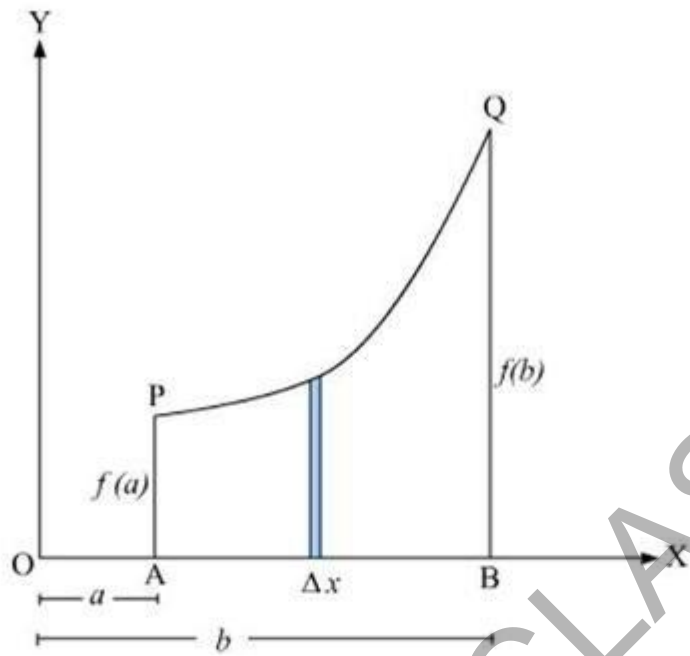
$$\text{Or, } \frac{d^2y}{dx^2} < 0 \text{ at maximum}$$

Similarly, at the minimum:

$$\frac{d^2y}{dx^2} < 0 \text{ at maximum}$$

Integral calculus

In the graph we have a curve PQ representing the relation between *x* and *y*. The equation of the curve is, $y = f(x)$



We shall find out the area under this curve. That is we need the area of APQB. To find this we shall first consider a very thin rectangle touching the curve and standing on the x-axis. The width of the rectangle is so that both the edges of the side near the curve actually touch the curve almost at the same point which means that Δx is so small that it tends to zero.

Area of this thin rectangle is = $f(x)\,\Delta x$

We shall take *n* such rectangles and fill the area. The area of APQB in the sum of the area of the rectangles. This may be written as

$$S = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x$$

This quantity is also denoted as,

$$S = \int_a^b f(x)\,dx$$

- **Scalar Quantities—** Physical quantities which only have magnitude, but no direction
  Example: speed, distance, current, work

- **Vector Quantities—** Physical quantities that have both magnitude and direction
  Example: Velocity, displacement, acceleration, force

- **Scalar quantities:** These are the physical quantities that are not affected by the change in the coordinate systems used to define them. They do not have any direction. Example: Speed, charge, temperature, etc.

- **Vector quantities:** These are physical quantities that have both direction and magnitude. They change with change in the coordinate systems used to define them. Example: Displacement, velocity, etc.

- **Position vector:** Position vector of a point in a coordinate system is the straight line that joins the origin and the point.

- **Displacement Vector:** It is the straight line that joins the initial and the final position.

- **Equality of Vectors:** Two vectors are said to be equal only if they have the same magnitude and the same direction.

- **Negative vector:** Negative vector is a vector whose magnitude is the same as that of a given vector, but whose direction is opposite to that of the given vector.

- **Zero vector:** Zero vector is a vector whose magnitude is zero and have an arbitrary direction.

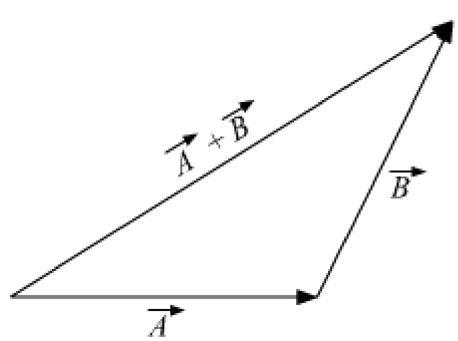
- **Resultant vector:** The resultant vector of two or more vectors is the vector which produces the same effect as produced by the individual vectors together.

- **Multiplication of Vectors by Real Numbers**

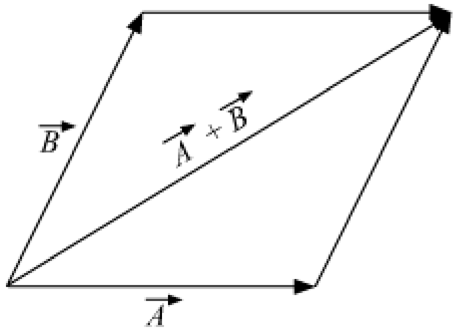
- Multiplication of a vector with a positive number k only changes the magnitude of the vector keeping its direction unchanged.

- Multiplication of a vector with a negative number $-k$ changes the magnitude and direction of the vector.

- **Addition of vectors:**
  - Head-to-tail/ triangle method



- Parallelogram method



- **Vector addition follows commutative and associative laws**:

  - A→+B→=B→+A→A→+B→=B→+A→ [Commutative]
  - (A→+B→)+C→=A→+(B→+C→)A→+B→+C→=A→+B→+C→ [Associative]

- **Subtraction of vector:**

  A→−B→=A→+(−B→)A→-B→=A→+-B→

- **Polygon law of vector addition:**

  - According to this law, if a number of vectors acting in a plane are represented in magnitudes and directions by the sides of an open polygon taken in order, then resultant vector is represented in magnitude and direction by the closing side of the polygon taken in the opposite order. The direction of the resultant vector is from the starting point of the first vector to the end point of the last vector.

- **Unit vector:** Unit vector is a vector of unit magnitude along the direction of the vector.

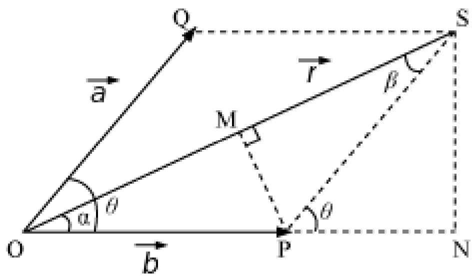
  aˆ=a→|a|,   a→=aˆ|a| a^=a→a,   a→=a^a

- In 2-D vector, a→a→ can be expressed as a→=axiˆ+ayjˆa→=axi^+ayj^
- If a→a→ makes θ angle with X axis, then

  $a_x = a \cos \theta$ and $a_y = a \sin \theta$

  - The same process is used to resolve a vector into three components along X-axis, Y-axis, and Z-axis.

- **Resultant of two vectors**



r→=a→+b→r→=a→+b→

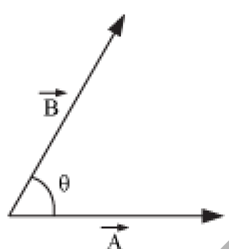
- **Law of cosines**

  ||r→||=||a→||+|||b→|||+2||a→|||||b→|||cos θ−−−−−−−−−−−−−−−−−−−−−√r→=a→+b→+2a→b→cos θ

- **Law of sines**

- Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by

  $\vec{A}.\vec{B} = AB\cos\theta$



  - The result of the scalar product of two vectors is a scalar quantity.
  - When two vectors are parallel their scalar product is equal to the product of their magnitudes.
  - When two vectors are perpendicular their scalar product is equal to zero.
  - **Properties of Scalar Product of two vectors**
    - Scalar product of two vectors is commutative, i.e.,

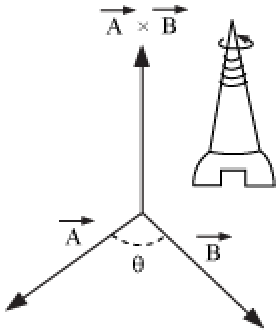
      $\vec{A}.\vec{B} = \vec{B}.\vec{A}$
    - Scalar product is distributive, i.e.,

      $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

- Scalar product of a vector with itself gives the square of its magnitude.
- Dot Product of two vectors $\vec{A}$ and $\vec{B}$ in Cartesian Coordinates is

  $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

- The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitude of the vectors $\vec{A}$ and $\vec{B}$ and sine of the smaller angle between them.



$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$

- The cross product of the two vectors is at right angles to both the vectors and points in the direction in which a right-handed screw will advance.

- Properties of vector product:

  - The cross product of a vector with itself is a null vector.
  - The cross product of two vectors does not obey commutative law. That is,

    $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

  - The cross product of vectors obeys the distributive law. That is,

    $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

- If the vectors A→A→ and B→B→ represent the two adjacent sides of a parallelogram, the magnitude of cross product of A→A→ and B→B→ will represent the area of the parallelogram.