Motion In A Straight Line

Differentiation

Differentiation is the process of obtaining the derived function f'(x) from the function f(x), where f'(x) is the derivative of f at x.

The derivatives of certain common functions are given in the Table of derivatives,

Table of derivatives :

$f(x)$	$f'(x)$
x^n	nx^{n--1}
sin x	cos x
cos x	−-sin x
tan x	sec^2 x
cot x	−-cosec^2 x
sec x	sec x tan x
cosec x	−-(cosec x)(cot x)
ln x	1x1x
e^x	e^x

Many other functions can be differentiated using the following rules of differentiation:

(i) If $h(x) = k\,f(x)$ for all x, where k is a constant, then $h'(x) = k\,f'(x)$.

(ii) If h(x) = $f(x) + g(x)$ for all x, then $h'\ (x) = f'(x) + g'(x)$.

(iii) The product rule: If $h(x) = f(x)g(x)$ for all x, then h'(x) = $f(x)g'$(x) + $f'(x)$+$g'(x)$.

(iv) The reciprocal rule: If $h(x) = 1/f(x)$ and $f(x) \neq 0$ for all x, thenh'(x) = −f'(x)(f(x))2h'(x) = -f'(x)(f(x))2

(v) The quotient rule: If $h(x) = f(x)/g(x)$ and $g(x) \neq 0$ for all x, then
h'(x) = g(x)f'(x)−f(x)g'(x)(g(x))2h'(x) = g(x)f'(x)-f(x)g'(x)(g(x))2

(vi) The chain rule: If $h(x) = (f \circ g)(x) = f(g(x))$ for all x, then $h'(x) = f'(g(x))g'(x)$.

Integration

Integration is the process of finding an anti-derivative of a given function f. 'Integrate f' means 'find an anti-derivative of f'. Such an anti-derivative may be called an indefinite integral of f and be denoted by ∫f(x)dx∫f(x)dx.

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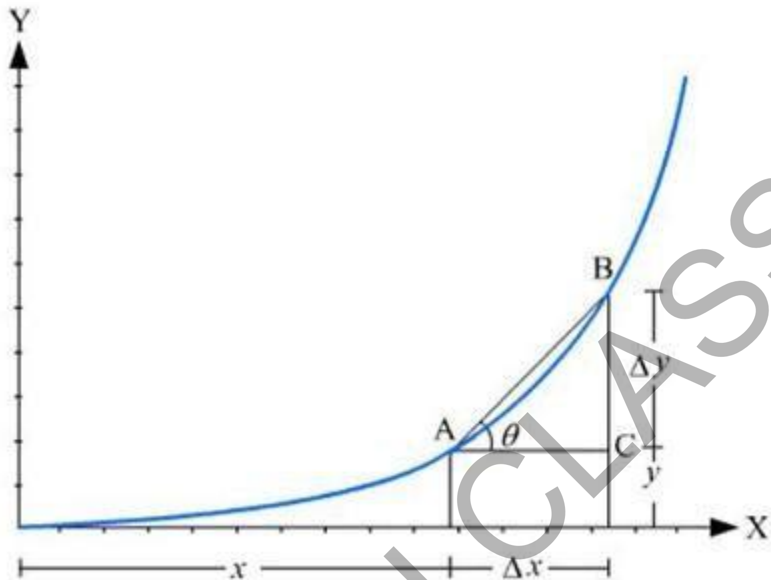
The term ‘integration’ is also used for any method of evaluating a definite integral. ∫baf(x)dx∫abf(x)dx

The definite integral can be evaluated if an anti-derivative Φ of *f* can be found, because then its value is Φ(*b*) − Φ(*a*). (This is provided that *a* and *b* both belong to an interval in which *f* is continuous.)

However, for many functions *f*, there is no anti-derivative expressible in terms of elementary functions, and other methods for evaluating the definite integral have to be sought, one such being so-called numerical integration.

Differential calculus



Let *x* and *y* be two quantities interrelated in such a way that for each value of *x* there is one and only one value of *y*.

The graph represents the *y* versus *x* curve. Any point in the graph gives an unique values of *x* and *y*. Let us consider the point A on the graph. We shall increase *x* by a small amount Δ*x*, and the corresponding change in *y* be Δ*y*.

Thus, when *x* change by Δ*x*, *y* change by Δ*y* and the rate of change of *y* with respect to *x* is equal to $\frac{\Delta y}{\Delta x}$

In the triangle ABC, coordinate of A is (x, y); coordinate of B is (*x* + Δ*x*, *y* + Δ*y*)

The rate $\frac{\Delta y}{\Delta x}$ can be written as,

$$\frac{\Delta y}{\Delta x}=\frac{BC}{AC}=\tan\theta=\text{slope of the line AB}$$

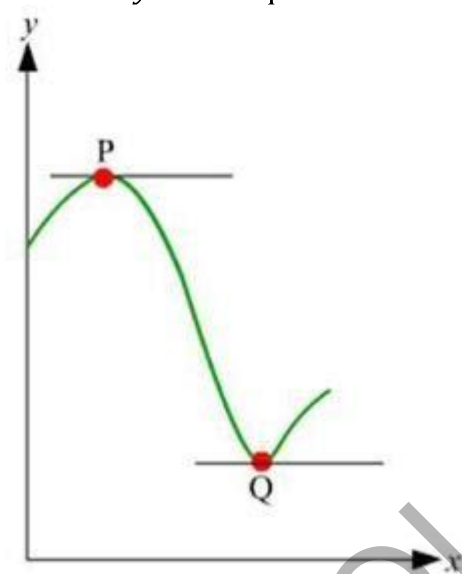
But this cannot be the precise definition of the rate because the rate also varies between the point A and B. So, we must take very small change in *x*. That is Δx is nearly equal to zero. As we make Δx smaller and smaller the slope $\tan\theta\tan\theta$ of the line AB approaches the slope of the tangent at A. This slope of the tangent at A gives the rate of change of *y* with respect to *x* at A.

This rate is denoted by $\frac{dy}{dx}$ and,

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

Maxima and Minima

Let *x* and *y* be two quantities interrelated in manner as shown in the graph below:



At the points P and Q the tangents to the curve is parallel to the x-axis.

Hence, its slope $\tan\theta\tan\theta = 0$.

But we know that the slope of the curve at any point equals the rate of change $\frac{dy}{dx}$ at the point.

Thus, at maximum (at P) or at minimum (at Q),

$$\frac{dy}{dx}=0$$

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. This implies, $\frac{dy}{dx}$ decrease at a maximum.

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$$\therefore \frac{d}{dx}\left(\frac{dy}{dx}\right) < 0 \text{ at maximum}$$

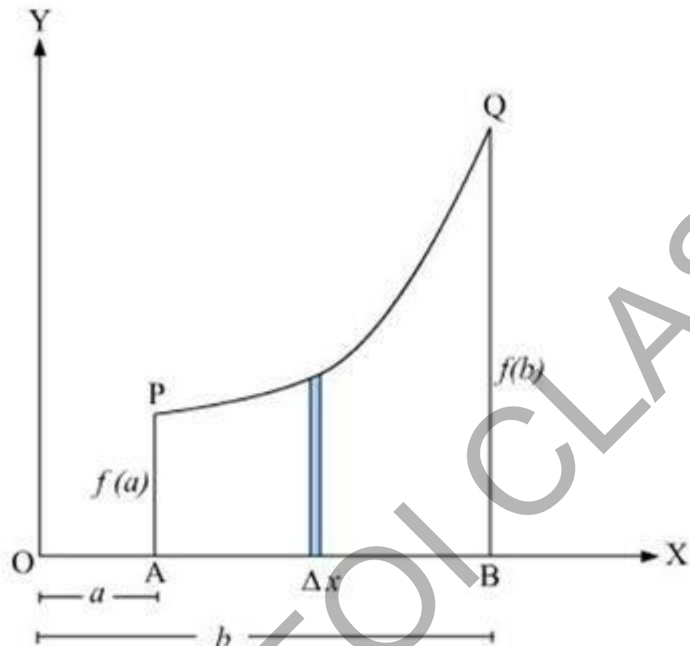
$$\text{Or, } \frac{d^2y}{dx^2} < 0 \text{ at maximum}$$

Similarly, at the minimum:

$$\frac{d^2y}{dx^2} < 0 \text{ at maximum}$$

Integral calculus

In the graph we have a curve PQ representing the relation between *x* and *y*. The equation of the curve is, $y = f(x)$



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We shall find out the area under this curve. That is we need the area of APQB. To find this we shall first consider a very thin rectangle touching the curve and standing on the x-axis. The width of the rectangle is so that both the edges of the side near the curve actually touch the curve almost at the same point which means that Δx is so small that it tends to zero.

Area of this thin rectangle is = $f(x)\,\Delta x$

We shall take *n* such rectangles and fill the area. The area of APQB in the sum of the area of the rectangles. This may be written as

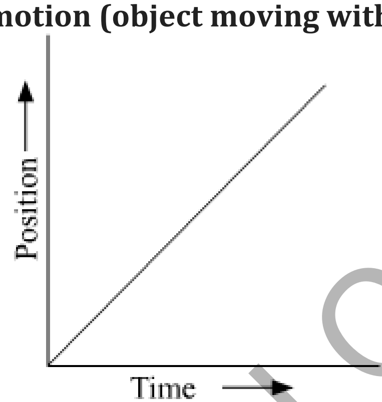
$$S = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x$$

This quantity is also denoted as,

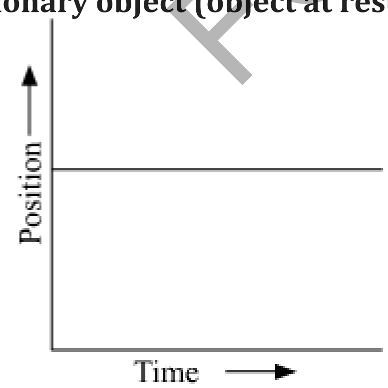
$$S = \int_a^b f(x)\,dx$$

- An object is at rest when the position of the object does not change with time and with respect to its surroundings.
- An object is in motion when the position of the object changes with time and with respect to its surroundings.
- Rest and motion are relative.
- If the distance covered by an object is much greater than its size during its motion, then the object is considered as point mass object.
- Distance or path length — Total length of the path covered by a body (scalar quantity)
- Displacement — Shortest distance between initial and final positions measured along a particular direction

- **Uniform motion (object moving with a constant velocity):**



- **Stationary object (object at rest)**:



- **Average velocity (slope of the *x*-*t* graph)**

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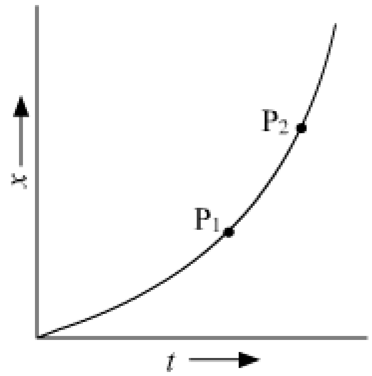
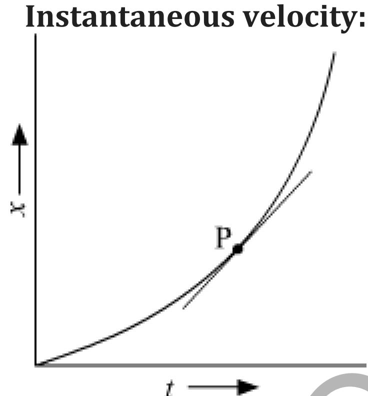


∴ Average velocity = slope of $\overline{P_1P_2}$

- **Average speed** = Total path lengthTotal time intervalTotal path lengthTotal time interval
  [No direction is considered]

- **Instantaneous velocity:**



= slope of the tangent at point P

- **Average acceleration:**

  a=v2−v1t2−t1=ΔvΔta=v2-v1t2-t1=ΔvΔt

- **Instantaneous acceleration**:

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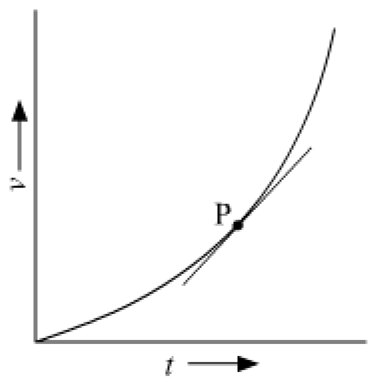
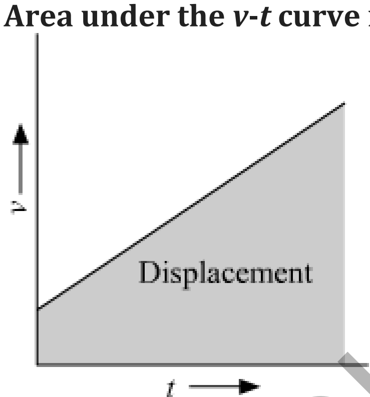


a=limΔt→0ΔvΔt=dvdta=limΔt→0ΔvΔt=dvdt = slope of the tangent at point P

- Velocity-time graph showing constant acceleration, increasing acceleration and decreasing acceleration:

- **Area under the *v*-*t* curve is equal to the displacement of the body.**



- **Equation of motion**

1st equation $v = u + at$

2nd equation
$s = ut + \frac{1}{2}at^2$

3rd equation $2as = v^2 - u^2$

Equations of motions (Kinematic equations) [When acceleration is uniform]

Velocity-time relation
v=u+atv=u+at
Disatnce-time relation
s=ut + 12at2s=ut + 12at2
Velocity-displacent relation

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v2−u2=2asv2-u2=2as

- Distance travelled in n^{th}second of uniformly accelerated motion is given by the relation,

$$D_n = u + \frac{a}{2}(2n-1)$$

Galileo's law of odd number

- The ratios of the distance covered by a body falling from the rest increase by odd numbers from one second to the next.That means, distances covered by each will increase by factors of 1, 3, 5, 7, …

Relative Velocity

- The relative velocity of a body **A** with respect to another body **B** (v_{AB}) is the time rate at which **A** changes its position with respect to **B.**
  - **Case 1: Both bodies move in the same direction**: If **A** and **B** are moving in the same direction, then the resultant relative velocity is $v_{AB} = v_A - v_B$
  - **Case 2: The bodies move in opposite directions**: If **A** and **B** are moving in the opposite directions, then the resultant relative velocity is $v_{AB} = v_A + v_B$

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