

JEE Mains 2018

Physics

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is :

- (1) 2.5%
- (2) 3.5%
- (3) 4.5%
- (4) 6%

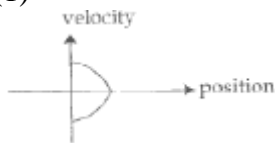
Solution:

$$\begin{aligned}\rho &= \frac{M}{L^3} \\ \frac{\Delta\rho}{\rho} &= \frac{\Delta M}{M} + \frac{3\Delta L}{L} \\ &= 1.5 + 3 \times 1 \\ &= 4.5\%\end{aligned}$$

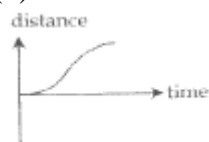
Hence the Solution is Option (3)

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

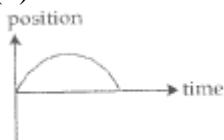
(1)



(2)



(3)



(4)



Solution:

Set: A

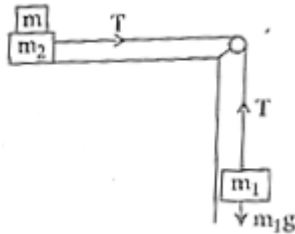
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(1),(3),(4) uniform retardation & then uniform acceleration.

(2) Normal uniform acceleration & retardation.

Hence the Solution is Option (2)

3. Two masses $m_1 = 5 \text{ kg}$ and $m_2 = 10 \text{ kg}$, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :



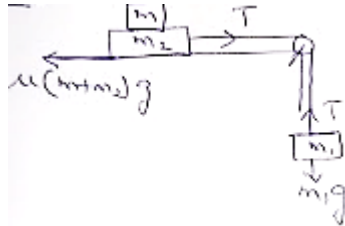
(1) 18.3 g

(2) 27.3 kg

(3) 43.3 kg

(4) 10.3 kg

Solution:



To stop the motion,

$$m_1 g = T = \mu(m_1 + m_2) g$$

$$\Rightarrow \frac{m_1}{\mu} - m_2 = m$$

$$\Rightarrow \frac{5}{0.15} - 10 = m$$

$$\Rightarrow m = 23.3 \text{ kg}$$

Hence the nearest Solution is Option (2)

4. A particle is moving in a circular path of radius a under the action of an attractive potential

$U = -\frac{k}{2r^2}$. Its total energy is :

(1) $-\frac{k}{4a^2}$

(2) $\frac{k}{2a^2}$

(3) Zero

(4) $-\frac{3}{2} \frac{k}{a^2}$

Set: A

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Solution:

$$u = \frac{-K}{2r^2}$$

$$\text{But } F = \frac{-du}{dr} = \frac{-K}{r^3}$$

‘ - ‘ sign of force implies attractive force

So,

$$\frac{K}{r^3} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{mv^2}{2} = \frac{K}{2r^2} = E_K$$

$$U_T = \frac{-K}{2r^2} + \frac{K}{2r^2}$$

$$= 0$$

Hence the Solution is Option (3)

5. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is :

(1) $\frac{v_0}{4}$

(2) $\sqrt{2}v_0$

(3) $\frac{v_0}{2}$

(4) $\frac{v_0}{\sqrt{2}}$

Solution:

By cons of linear momentum,

$$mv_0 = mv_1 + mv_2$$

$$v_0 = v_1 + v_2 \dots (i)$$

$$\text{Also, } \frac{\left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2\right) - \left(\frac{1}{2}mv_0^2\right)}{\frac{1}{2}mv_0^2} \times 100$$

$$= 50$$

$$\Rightarrow v_1^2 + v_2^2 - v_0^2 = \frac{1}{2}v_0^2$$

$$\Rightarrow v_1^2 + v_2^2 = \frac{3}{2}v_0^2$$

Using (i)

Set: A

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$$(v_0 - v_2)^2 + v_2^2 = \frac{3}{2} v_0^2$$

$$v_2^2 - 2v_0v_2 + v_1^2 = \frac{1}{2} v_0^2$$

$$2v_2^2 + (-2v_0)v_2 + \left(\frac{-1}{2}v_0^2\right) = 0$$

$$v_2 = \frac{2v_0 \pm \sqrt{4v_0^2 + 4v_0^2}}{4}$$

$$= \frac{v_0 \pm \sqrt{2}v_0}{2}$$

$$\text{If } v_2 = \frac{v_0 \pm \sqrt{2}v_0}{2}$$

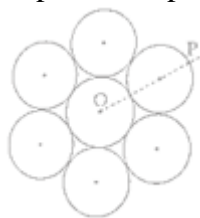
$$\Rightarrow v_1 = \frac{v_0 \pm \sqrt{2}v_0}{2}$$

So relative velocity :

$$\Rightarrow v_2 - v_1 = \sqrt{2}v_0$$

Hence the Solution is Option (2)

6. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is :



(1) $\frac{19}{2}MR^2$

(2) $\frac{55}{2}MR^2$

(3) $\frac{73}{2}MR^2$

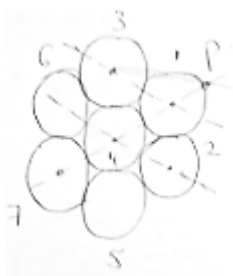
(4) $\frac{181}{2}MR^2$

Solution:

Moment of inertia of each disc about the given axis is,

Set: A

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$$I_1 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$I_4 = \frac{1}{2}MR^2 + M(3R)^2 = \frac{19}{2}MR^2$$

$$I_7 = \frac{1}{2}MR^2 + M(3R)^2 = \frac{19}{2}MR^2$$

$$I_{net} = I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7$$

$$I_{net} > I_1 + I_4 + I_7$$

$$I_{net} > \frac{73}{2}MR^2$$

Hence the Solution is Option (3)

7. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is :



(1) $4MR^2$

(2) $\frac{40}{9}MR^2$

(3) $10MR^2$

(4) $\frac{37}{9}MR^2$

Solution:

Mass of disc = Volume \times Density

$$9M = A \times T \times \rho \text{ (Area \times thickness \times density)}$$

$$9M = \pi R^2 \times t \times \rho \dots\dots (i)$$

For the disc which is cut off.

$$M' = \pi \left(\frac{R}{3} \right)^2 \times t \times \rho \dots\dots (ii)$$

Set: A

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(i) Divided by (ii)

$$\frac{9M}{M} = 9 \Rightarrow M' = M$$

Moment of inertia of complete disc about an axis passes through O.

$$I_1 = \frac{1}{2}(9M) \times R^2$$

Moment of inertia of cut off disc about an axis passes through O

$$\begin{aligned} I_2 &= \frac{1}{2}M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2 \\ &= \frac{1}{2} \frac{MR^2}{9} + \frac{4MR^2}{9} \end{aligned}$$

So, moment of inertia of remaining disc = $I_1 - I_2$

$$\begin{aligned} &= \frac{9MR^2}{2} - \frac{MR^2}{18} - \frac{4MR^2}{9} \\ &= \left(\frac{81-1-8}{18}\right)MR^2 \\ &= 4MR^2 \end{aligned}$$

Hence the Solution is Option (1)

8. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then :

(1) $T \propto R^{3/2}$ for any n.

(2) $T \propto R^{\frac{n}{2}+1}$

(3) $T \propto R^{(n+1)/2}$

(4) $T \propto R^{n/2}$

Solution:

$$F = KR^{n-1}$$

$$\text{So, } \frac{mv^2}{R} = \frac{K}{R^n}$$

$$\Rightarrow v = \sqrt{\frac{K}{M}} \times \frac{1}{R^{(n-1)/2}}$$

$$\text{Now, } T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\frac{K}{M}}} \times R^{\frac{n-1}{2}}$$

$$\Rightarrow T \propto R^{(n+1)/2}$$

Hence the Solution is Option (3)

9. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the

Set: A

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liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$ is :

- (1) $\frac{Ka}{mg}$
- (2) $\frac{Ka}{3mg}$
- (3) $\frac{mg}{3Ka}$
- (4) $\frac{mg}{Ka}$

Solution: Bulk Modulus,



$$K = \frac{\Delta P}{(\Delta V/V)}$$

$$\text{For sphere, } V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3\Delta r}{r}$$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\Delta P}{3K} = \frac{Mg}{3Ka}$$

Hence the Solution is Option (3)

10. Two moles of an ideal monoatomic gas occupies a volume V at 27°C . The gas expands adiabatically to a volume $2V$. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

- (1) (a) 189 K (b) 2.7 kJ
- (2) (a) 195 K (b) -2.7 kJ
- (3) (a) 189 K (b) -2.7 kJ
- (4) (a) 195 K (b) 2.7 kJ

Solution:

Initially $n = 2$, v , $T = 300\text{K}$

Finally $V_d = 2v$

Gas is monoatomic, So, $r = 5/3$

So,

Set: A

$$d_w = \frac{nR(T_i - T_f)}{r - 1}$$

$$T_i V_i^{r-1} = T_f V_f^{r-1}$$

$$T_i \times \left(\frac{1}{2}\right)^{2/3} = T_f$$

$$\Rightarrow T_f = \frac{300}{4^{1/3}} = \frac{300}{1.6} \approx 189\text{K}$$

Since gas undergoes expansion.

$$\text{So, } d_w > 0$$

$$\& d_q = d_u + d_w$$

$$\Rightarrow d_u < 0.$$

Hence the Solution is Option (3)

11. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s , then the pressure on the wall is nearly :

(1) $2.35 \times 10^3 \text{ N/m}^2$

(2) $4.70 \times 10^3 \text{ N/m}^2$

(3) $2.35 \times 10^2 \text{ N/m}^2$

(4) $4.70 \times 10^2 \text{ N/m}^2$

Solution:

Change in momentum normal to the wall



$$= 2mv \cos 45^\circ$$

$$\text{So, Force} = \frac{(2mv \cos 45^\circ) \times N}{t}$$

$$\text{Pressure} = \frac{2 \times 3.32 \times 10^{-27} \times 10^3 \times \frac{1}{\sqrt{2}} \times 10^{23}}{2 \times 10^{-4} \times 1}$$

$$P = 2.35 \times 10^3 \text{ N/m}^2$$

Hence the Solution is Option (1)

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} /sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver = 108 and Avagadro number = $6.02 \times 10^{23} \text{ gm mole}^{-1}$)

(1) 6.44 N/m

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- (2) 7.1 N/m
- (3) 2.2 N/m
- (4) 5.5 N/m

Solution:

$$\begin{aligned}T &= 2\pi\sqrt{\frac{M}{K}} \\K &= \frac{4\pi^2 \times M}{T^2} \\&= 4\pi^2 \times \frac{108 \times 10^{-3}}{6.023 \times 10^{-23}} \times (10^{12})^2 \\&= 4\pi^2 \times \frac{10.8}{6.023} \times 10^{-2} \\&= 7.1 \text{ N / m}\end{aligned}$$

Hence the Solution is Option (2)

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3 \text{ kg/m}^3$ and its young's modulus is $9.27 \times 10^{10} \text{ Pa}$. What will be the fundamental frequency of the longitudinal vibrations ?

- (1) 5 kHz
- (2) 2.5 kHz
- (3) 10 kHz
- (4) 7.5 kHz

Solution:

Since rod is clamped at centre. So centre it behaves as node & end it behave as antinode. So,

$$\begin{aligned}\frac{\lambda}{4} &= 30 \text{ cm} \\ \Rightarrow \lambda &= 1.2 \text{ m} \\ v &= \sqrt{\frac{y}{\rho}} = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}} \\ &= 5.85 \times 10^3 \text{ m / s} \\ \text{So, } v &= \frac{5.85 \times 10^3}{1.2} \\ &= 5 \text{ kHz}\end{aligned}$$

Hence the Solution is Option (1)

14. Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The potential of shell B is :

(1) $\frac{\sigma}{\epsilon_0} \left(\frac{a^2 - b^2}{a} + c \right)$

Set: A

$$(2) \frac{\sigma}{\epsilon_0} \left(\frac{a^2 - b^2}{b} + c \right)$$

$$(3) \frac{\sigma}{\epsilon_0} \left(\frac{b^2 - c^2}{b} + a \right)$$

$$(4) \frac{\sigma}{\epsilon_0} \left(\frac{b^2 - c^2}{c} + a \right)$$

Solution:

Charge in sphere

$$A = q_A = \sigma \times 4\pi a^2$$

$$B = q_B = \sigma \times 4\pi b^2$$

$$C = q_C = \sigma \times 4\pi c^2$$

Potential of B:

$$= \frac{q_A}{4\pi \epsilon_0 b} + \frac{q_b}{4\pi \epsilon_0 b} + \frac{q_c}{4\pi \epsilon_0 c}$$

$$= \frac{\sigma a^2}{\epsilon_0 b} + \frac{-\sigma b^2}{\epsilon_0 b} + \frac{\sigma c^2}{\epsilon_0 c}$$

$$= \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right)$$

$$= \frac{\sigma}{\epsilon_0} \left(\frac{a^2 - b^2}{b} + c \right)$$

Hence the Solution is Option (2)

15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the

induced charge will be :

(1) 1.2 n C

(2) 0.3 n C

(3) 2.4 n C

(4) 0.9 n C

Solution:

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$$\begin{aligned}q_i &= C_0 V \\&= 90 \times 10^{-12} \times 20 \\&= 1800 \times 10^{-12} \\&= 1.8 nC\end{aligned}$$

$$\begin{aligned}q_f &= \frac{5}{3} \times 1.8 nC \\&= 3 nC\end{aligned}$$

$$\begin{aligned}\text{So, } q_{ind} &= 3 nC - 1.8 nC \\&= 1.2 nC\end{aligned}$$

Hence the Solution is Option (1)

16. In an a.c. circuit, the instantaneous e.m.f. and current are given by $e = 100 \sin 30 t$

$i = 20 \sin \left(30 t - \frac{\pi}{4} \right)$. In one cycle of a.c., the average power consumed by the circuit and the

wattless current are, respectively :

(1) 50, 10

(2) $\frac{1000}{\sqrt{2}}$, 10

(3) $\frac{50}{\sqrt{2}}$, 0

(4) 50, 0

Solution:

$$\begin{aligned}P_{qv} &= P_{rms} \times I_{rms} \times \cos \phi \\&= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos \left(\frac{\pi}{4} \right) \\&= \frac{2000}{2\sqrt{2}} \text{ watt} \\&= \frac{1000}{\sqrt{2}} \text{ w} \\I_w &= I_{rms} \times \cos \phi \\&= \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \\&= 10 A\end{aligned}$$

Hence the Solution is Option (2)

17. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10Ω . The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between :

(1) 11.6 V and 11.7 V

(2) 11.5 V and 11.6 V

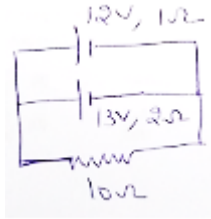
(3) 11.4 V and 11.5 V

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(4) 11.7 V and 11.8 V

Solution:



Total

$$\begin{aligned}
 emf &= \frac{\left(\frac{\rho_1}{r_1} + \frac{\rho_2}{r_2} \right)}{\left(\frac{1}{r_1} + \frac{1}{r_2} \right)} \\
 &= \frac{\left(\frac{12}{1} + \frac{13}{2} \right)}{\left(\frac{1}{1} + \frac{1}{2} \right)} \\
 &= \frac{37}{3} \text{ Volt} \\
 \text{So, } V &= \rho - Ir \\
 &= \rho - \frac{\rho r}{R + r} \\
 &= 11.53V
 \end{aligned}$$

Hence the Solution is Option (2)

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e , r_p , r_α respectively in a uniform magnetic field B. The relation between r_e , r_p , r_α is :

- (1) $r_e > r_p = r_\alpha$
- (2) $r_e < r_p = r_\alpha$
- (3) $r_e < r_p < r_\alpha$
- (4) $r_e < r_\alpha < r_p$

Solution:

For charge moving in circular orbit is a uniform magnetic field,

$$\begin{aligned}
 r &= \frac{mv}{Bq} = \frac{\sqrt{2mE_K}}{Bq} \\
 \Rightarrow E_K &= \frac{B^2 q^2 r^2}{2m}
 \end{aligned}$$

Since all particles have same E_K & B ,

So,

Set: A

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$$\frac{q^2 r^2}{2m} = \text{Constant}$$

$$q_e = q_p \text{ \& } m_e < m_p$$

$$\Rightarrow r_e < r_p$$

For proton & α -particle

$$\frac{12 \times r_p^2}{2 \times 1} = \frac{22 \times r_\alpha^2}{2 \times 4}$$

$$\Rightarrow r_p = r_\alpha$$

Hence the Solution is Option (2)

19. The dipole moment of a circular loop carrying a current I , is m and the magnetic field at the centre of the loop is B_1 . When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2 . The ratio $\frac{B_1}{B_2}$ is :

(1) 2

(2) $\sqrt{3}$

(3) $\sqrt{2}$

(4) $\frac{1}{\sqrt{2}}$

Solution:

Magnetic moment, $M = IA = I \times \pi r^2$ and magnetic field at the centre of circle

$$= B = \frac{\mu_0 I}{2R}$$

$$M_f = I \times \pi r_f^2 = 2M$$

$$M_i = I \times \pi r_i^2 = M$$

$$\Rightarrow \frac{r_f}{r_i} = \sqrt{2}$$

$$\text{So, } \frac{B_1}{B_2} = \frac{r_f}{r_i} = \sqrt{2}$$

Hence the Solution is Option (3)

20. For an RLC circuit driven with voltage of amplitude V_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by :

(1) $\frac{\omega_0 L}{R}$

Set: A

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- (2) $\frac{\omega_0 R}{L}$
(3) $\frac{R}{(\omega_0 C)}$
(4) $\frac{CR}{\omega_0}$

Solution:

$Q_{\text{factor}} = (\text{voltage across L or C at Resonance})/(\text{voltage across R})$

$$= \frac{I \times X_L}{I \times R} = \frac{\omega_0 L}{R}$$

Hence the Solution is Option (1)

21. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01} \hat{x} \cos \left[2\pi \nu \left(\frac{z}{c} - t \right) \right]$ in air and $\vec{E}_2 = E_{02} \hat{x} \cos [k(2z - ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct ?

- (1) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$
(2) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$
(3) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$
(4) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$

Solution:

$$\vec{E}_1 = E_{01} \hat{x} \cos \left[2\pi \nu \left(\frac{z}{c} - t \right) \right]$$
$$\vec{E}_2 = E_{02} \hat{x} \cos [k(2z - ct)]$$

Where

$$k = \frac{2\pi}{\lambda} \quad \& \quad \frac{\nu}{c} = \frac{1}{\lambda}$$

So volume in medium 1 = C

Volume in medium 2 = C/2

$$C = \frac{1}{\sqrt{\mu \epsilon_0 \epsilon_{r_1}}}$$

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$$\frac{C}{2} = \frac{1}{\sqrt{\mu \epsilon_0 \epsilon_{r_2}}}$$

$$\Rightarrow 2 = \sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}$$

$$\Rightarrow \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$$

Hence the Solution is Option (3)

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is :

- (1) 0°
- (2) 30°
- (3) 45°
- (4) 60°

Solution:

Unpolarized light of intensity I , when passed through a polarizer A, its intensity becomes $\frac{I}{2}$

Since intensity of light emerging from polarizer B = $\frac{I}{2}$

So, A & B are parallel placed.

Let, C makes angle θ with A.

So,

$$\frac{I}{2} \cos^4 \theta = \frac{I}{8}$$

$$\Rightarrow \cos^2 \theta = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

Hence the Solution is Option (3)

23. The angular width of the central maximum in a single slit diffraction pattern is 60° . The width of the slit is $1 \mu\text{m}$. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a

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distance of 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance ?

(i.e. distance between the centres of each slit.)

- (1) $25\mu\text{m}$
- (2) $50\mu\text{m}$
- (3) $75\mu\text{m}$
- (4) $100\mu\text{m}$

Solution:

$$\frac{2\lambda}{d} = 60^\circ \times \frac{\pi}{180}$$

$$\lambda = \left(\frac{\pi}{3} \times \frac{1}{2} \right) \mu\text{m}$$

In Ydse,

$$B' = \frac{\lambda D}{d'} \times 10^{-6}$$

$$\Rightarrow 10^{-2} = \frac{\left(\frac{\pi}{2} \times \frac{1}{2} \right) \times \frac{1}{2}}{d'} \times 10^{-6}$$

$$\Rightarrow d' = \frac{\pi}{10^{-2} \times 12} \times 10^{-6}$$

$$= 26.16 \approx 25\mu\text{m}$$

Hence the Solution is Option (1)

24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n , (A, B are constants)

- (1) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$
- (2) $\Lambda_n \approx A + B\lambda_n$
- (3) $\Lambda_n^2 \approx A + B\lambda_n^2$
- (4) $\Lambda_n^2 \approx \lambda$

Solution:

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}}$$

$$E_k = \frac{h^2}{2m\lambda^2}$$

For emitted photon,

$$E_n - E_1$$

$$= \frac{hc}{\Lambda_n}$$

$$\Rightarrow \frac{h^2}{2m\lambda_n^2} - \frac{h^2}{2m\lambda_g^2} = \frac{hc}{\Lambda_n}$$

$$\Rightarrow \frac{hc}{\Lambda_n} = \frac{h^2}{2m\lambda_n^2} + K \quad \left[K = \frac{-h^2}{2m\lambda_g^2} \right]$$

$$\Rightarrow \Lambda_n^2 = A + B\lambda_n^2$$

Hence the Solution is Option (3)

25. If the series limit frequency of the Lyman series is ν_L , then the series limit frequency of the Pfund series is :

(1) $25\nu_L$

(2) $16\nu_L$

(3) $\frac{\nu_L}{16}$

(4) $\frac{\nu_L}{25}$

Solution:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \propto \nu$$

$$R \left(\frac{1}{1^2} \right) \propto \nu_l \quad \& \quad R \left(\frac{1}{5^2} \right) \propto \nu_p$$

$$\Rightarrow \frac{\nu_l}{\nu_p} = \frac{1}{25}$$

$$\Rightarrow \nu_p = \frac{\nu_l}{25}$$

Hence the Solution is Option (4)

26. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c . The values of p_d and p_c are respectively :

(1) (.89, .28)

(2) (.28, .89)

Set: A

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(3) (0, 0)

(4) (0, 1)

Solution: $U_1, U_2=0$

Since collision is elastic,

$$v_1 = \frac{u_1(M_1 - M_2) + 2M_2u_2}{M_1 + M_2}$$

But, $u_2 = 0$

$$\Rightarrow \frac{v_1}{u_1} = \frac{M_1 - M_2}{M_1 + M_2} \dots\dots\dots (i)$$

Fractional loss in kinetic energy of neutron,

$$= \frac{\frac{1}{2}M_1u_1^2 - \frac{1}{2}M_1v_1^2}{\frac{1}{2}M_1u_1^2}$$

$$= 1 - \left(\frac{v_1}{u_1}\right)^2$$

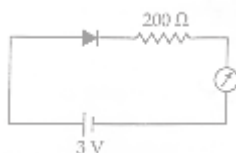
So, A.T.Q;

$$P_d = 1 - \left(\frac{1-2}{1+2}\right)^2 = \frac{8}{9} = 0.89$$

$$P_c = 1 - \left(\frac{1-12}{1+12}\right)^2 = 0.28$$

Hence the Solution is Option (1)

27. The reading of the ammeter for a silicon diode in the given circuit is :



(1) 0

(2) 15 mA

(3) 11.5 mA

(4) 13.5 mA

Solution:

The given diode is a forward bias & hence behaves as a perfect conductor i.e., it offers zero resistance.

So,

$$I = \frac{V}{R} = \frac{3}{200} = 1 \times 10^{-2} \text{ A}$$

$$= 15 \text{ mA}$$

Set: A

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Hence the Solution is Option (2)

28. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz ?

- (1) 2×10^3
- (2) 2×10^4
- (3) 2×10^5
- (4) 2×10^6

Solution:

Required number of channels

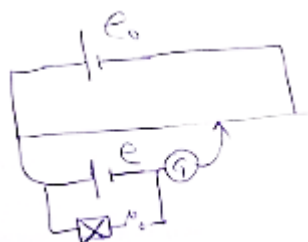
$$\begin{aligned} &= \frac{\frac{10}{100} \times 10 \times 10^9}{5 \times 10^3} \\ &= 2 \times 10^5 \end{aligned}$$

Hence the Solution is Option (3)

29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5Ω , a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

- (1) 1Ω
- (2) 1.5Ω
- (3) 2Ω
- (4) 2.5Ω

Solution:



$$\begin{aligned} r &= \left(\frac{l_1}{l_2} - 1 \right) R \\ &= \left(\frac{52}{40} - 1 \right) \times 5 \\ &= \frac{12 \times 5}{40} \\ &= 1.5\Omega \end{aligned}$$

Hence the Solution is Option (2)

Set: A

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30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combinations is $1\text{ k}\Omega$. How much was the resistance on the left slot before interchanging the resistances ?

- (1) 990Ω
- (2) 505Ω
- (3) 550Ω
- (4) 910Ω

Solution:



$$\frac{P}{Q} = \frac{l}{100-l}$$

$$\& \frac{Q}{P} = \frac{l-10}{110-l}$$

$$\Rightarrow \frac{l}{100-l} = \frac{110-l}{l-10}$$

$$\Rightarrow l^2 - 10l = 11000 - 100l - 100l + l^2$$

$$\Rightarrow 200l = 11000$$

$$\Rightarrow l = 55\text{cm}$$

$$\frac{\rho l}{A} + \frac{\rho(100-l)}{A} = 1000\Omega$$

$$\Rightarrow \frac{\rho}{A} = 10$$

$$\text{So, } \frac{\rho l}{A} = 10 \times 55$$

$$= 550\Omega$$

Hence the Solution is Option (3)

Chemistry

31. The ratio of mass present of C and H of an organic compound ($C_xH_yO_z$) is 6 : 1. If one molecule of the above compound ($C_xH_yO_z$) contains half as much oxygen as required to burn one molecule of compound C_xH_y completely to CO_2 and H_2O . The empirical formula of compound $C_xH_yO_z$ is :

- (1) $C_3H_6O_3$
- (2) C_2H_4O
- (3) $C_3H_4O_2$
- (4) $C_2H_4O_3$

Set: A

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Solution:

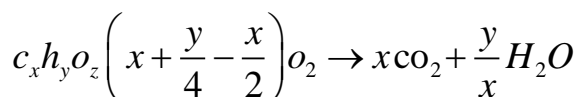
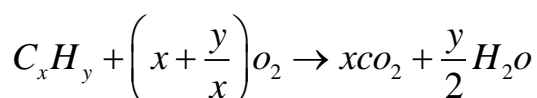
$C_xH_yO_z$ – Organic compound

$$\frac{\text{mass of } C}{\text{mass of } H} = 6:1$$

$$\frac{\text{mass of } C / 12}{\text{mass of } H / 1} = \frac{6}{12} : \frac{1}{2}$$

$$\therefore \frac{\text{No. of moles of } C}{\text{No. of moles of } H} = \frac{1}{2} : 1$$

$$C_xH_yO_z = \frac{\text{moles of } C}{\text{Moles of } H}$$



$$\frac{1}{2} \left(x + \frac{y}{4}\right) = x + \frac{y}{4} - \frac{z}{2}$$

$$\frac{x+y}{4} = 2x + \frac{y}{2} - Z$$

$$Z = x + \frac{y}{4}$$

$$c_2h_4o_3 \quad Z = 2 + \frac{4}{4} = 3$$

Hence the answer is option (4).

32. Which type of 'defect' has the presence of cations in the interstitial sites ?

- (1) Schottky defect
- (2) Vacancy defect
- (3) Frenkel defect
- (4) Metal deficiency defect

Solution:

Frenkel defect has the presence of cation in interstitial site.

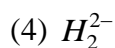
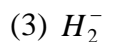
Hence the answer is option (3).

33. According to molecular orbital theory, which of the following will not be a viable molecule ?

- (1) H_2^{2+}
- (2) He_2^+

Set: A

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Solution :

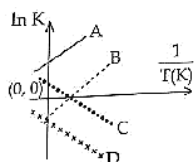
H_2^{2-} electronic configuration $\sigma 1s^2, \sigma^* 1s^2$

$$\text{Number of electrons} = 4 ; Bo = \frac{1}{2}(N_b - N_a) = \frac{1}{2}(2 - 2) = 0$$

Molecule does not exist.

Hence the answer is option (4).

34. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction?



(1) A and B

(2) B and C

(3) C and D

(4) A and D

Solution:

$$\ln K = \frac{-\Delta H^\circ}{RT} + \frac{\Delta S^\circ}{R} \quad (\text{Relation between Rate constant})$$

For exothermic reaction $\Delta H = -ve$.

$$\text{Slope} = \frac{-\Delta H^\circ}{R} > 0$$

Hence the answer is option (1).

35. The combustion of benzene (1) gives $CO_2(g)$ and $H_2O(l)$. Given that heat of combustion of benzene at constant volume is $-3263.9 \text{ kJ mol}^{-1}$ at 25°C ; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be :

(1) 4152.6

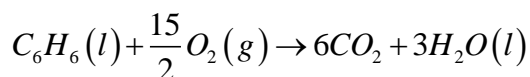
(2) -452.46

(3) 3260

(4) -3267.6

Solution:

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$$\Delta n_g = 6 - 7.5 = -1.5$$

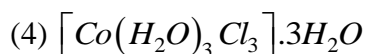
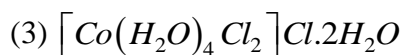
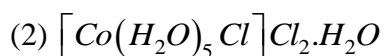
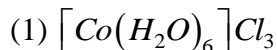
$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 - 1.5 \times 8.134 \times 10^{-3} \times 298$$

$$= -3267.6$$

Hence the answer is option (4).

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

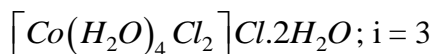
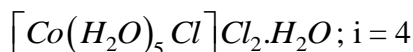
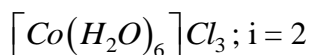
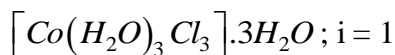


Solution:

$$\Delta T_f = i \times k_f \times m$$

$$\Delta T_f \propto i$$

$$T_f \propto \frac{1}{i}$$



Hence the answer is option (4).

37. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl . If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is :

(1) 5×10^{-8}

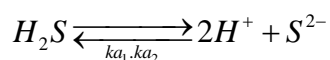
(2) 3×10^{-20}

(3) 6×10^{-21}

(4) 5×10^{-19}

Solution:

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$$0.1 - x \qquad 0.2 \quad x$$

$$k_{a_1}.k_{a_2} = \frac{(0.2)^2 \times 5^2}{0.1}$$

$$\frac{1.2 \times 10^{-2} \times 0.1}{0.04} = [S^{2-}]$$

$$[S^{2-}] = 3 \times 10^{-20}$$

Hence the answer is option (2).

38. An aqueous solution contains an unknown concentration of Ba^{2+} . When 50 mL of a 1M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10} . What is the original concentration of Ba^{2+} ?

- (1) 5×10^{-9} M
- (2) 2×10^{-9} M
- (3) 1.1×10^{-9} M
- (4) 1.0×10^{-10} M

Solution:

$$K_{sp}(BaSO_4) = 10^{-10}$$

$$Q_{sp} = K_{sp}$$

$$(Ba^{2+})(SO_4^{2-}) = 10^{-10}$$

$$Ba^{2+} \frac{50}{500} = 10^{-10}$$

$$\therefore (Ba^{2+}) = 10^{-9}$$

$$m_1 v_1 = m_2 v_2$$

$$m \times 450 = 10^{-9} \times 500$$

$$\therefore m = \frac{500}{450} \times 10^{-9}$$

$$m = 1.1 \times 10^{-9}$$

Hence the answer is option (3).

39. At 518°C , the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :

- (1) 2
- (2) 3
- (3) 1
- (4) 0

Solution:

Set: A

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$$r = k(p)^m \quad m = \text{order of reaction}$$

$$1 = k \left[363 \times \frac{95}{100} \right]^m \dots\dots\dots(1)$$

$$0.5 = k \left(362 \times \frac{67}{100} \right) \dots\dots\dots(2)$$

equation – 1 / Equation – 2

$$\frac{1}{0.5} = \left[\frac{95}{67} \right]^m$$

$$2 = \left(\frac{95}{67} \right)^m$$

$$2 = (0.4)^m$$

$$\log 2 = m \log 1.4$$

$$\frac{0.3010}{\log 1.4} = m$$

$$m \approx \frac{0.3010}{0.15}$$

$$m = 2$$

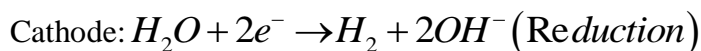
Hence the answer is option (1).

40. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane?

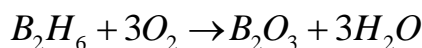
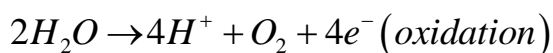
(Atomic weight of B = 10.8 u)

- (1) 6.4 hours
- (2) 0.8 hours
- (3) 3.2 hours
- (4) 1.6 hours

Solution:



Anode:



27.66 g

$$n = \frac{27.66}{276} \approx 1$$

Moles of O₂ required = 3

Set: A

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Gram equivalent of $O_2 = 3 \times 4 = 12$

Gram equivalent

$$w = \frac{it}{96500}$$

$$12 = \frac{100 \times t}{96500}$$

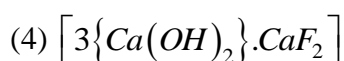
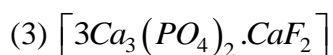
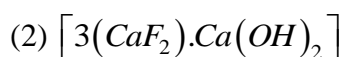
$$t = \frac{12 \times 965}{60 \times 60} \text{ sec}$$

$$t = 3.2 \text{ hr}$$

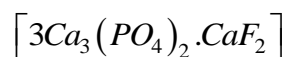
Hence the answer is option (3).

41. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$

:



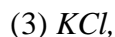
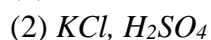
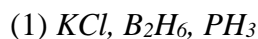
Solution:



Hence the answer is option (3).

42. Which of the following compounds contain(s) no covalent bond(s) ?

KCl , PH_3 , O_2 , B_2H_6 , H_2SO_4

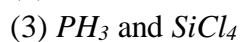
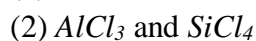
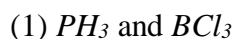


Solution:

KCl is an ionic compound. It cannot form covalent bond. Elements of s – block & p – block combine to form ionic compounds.

Hence the answer is option (3).

43. Which of the following are Lewis acids ?



Set: A

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(4) BCl_3 and $AlCl_3$

Solution:

BCl_3 and $AlCl_3$ are electron deficient compounds. Boron and aluminium has 6 electrons in their valence shell in BCl_3 & $AlCl_3$,. PH_3 , $SiCl_4$ have 8 electrons in their valence shell. They are not Lewis acids.

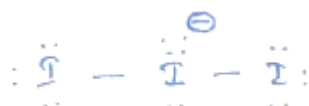
Hence the answer is option (4).

44. Total number of lone pair of electrons in I_3^- ion is :

- (1) 3
- (2) 6
- (3) 9
- (4) 12

Solution:

The total number of lone pair of electrons is I_3^- is 9



Hence the answer is option (1).

45. Which of the following salts is the most basic in aqueous solution?

- (1) $Al(CN)_3$
- (2) CH_3COOK
- (3) $FeCl_3$
- (4) $Pb(CH_3COO)_2$

Solution: CH_3COOK is a salt of weak acid and strong base

Hydrolysis of potassium acetate gives strong base KOH

Hence the answer is option (2).

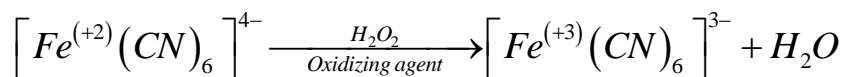
46. Hydrogen peroxide oxides $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively:

- (1) $(H_2O + O_2)$ and H_2O
- (2) $(H_2O + O_2)$ and $(H_2O + OH^-)$
- (3) H_2O and $(H_2O + O_2)$
- (4) H_2O and $(H_2O + OH^-)$

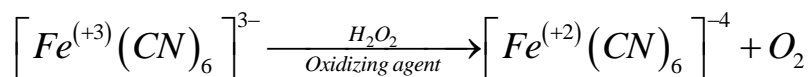
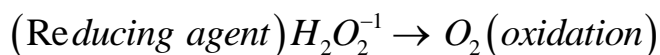
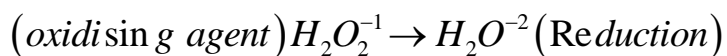
Set: A

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Solution:



reducing agent



(oxidizing agent)

Hence the answer is option (3).

47. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and

$K_2[Cr(CN)_2(O)_2(NH_3)]$ respectively are :

(1) +3, +4, and +6

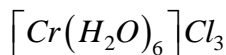
(2) +3, +2, and +4

(3) +3, 0, and +6

(4) +3, 0, and +4

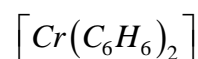
Solution:

The Oxidation state of Cr in



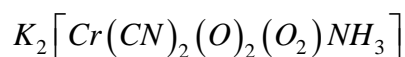
$$x + 6(0) - 3 = 0 \quad .$$

$$x = +3$$



$$x + 2(0) = 0$$

$$x = 0$$



$$+2 + x + 2(-1) + 2(-2) + 1(-2) + 0$$

$$= 6$$

$$\therefore x = +6$$

Hence the answer is option (3).

48. The compound that does not produce nitrogen gas by the thermal decomposition is :

(1) $Ba(N_3)_2$

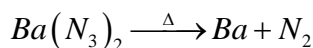
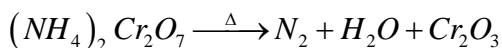
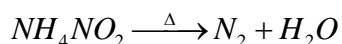
(2) $(NH_4)_2Cr_2O_7$

Set: A

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- (3) NH_4NO_2
(4) $(NH_4)_2SO_4$

Solution:



Hence the answer is option (4).

49. When metal 'M' is treated with $NaOH$, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of $NaOH$. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :

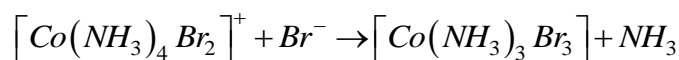
- (1) Zn
(2) Ca
(3) Al
(4) Fe

Solution:

The Gelatinous precipitate formed in $Al(OH)_3$; $Al(OH)_3$ on strong heating gives Al_2O_3 which is used in chromatography as an adsorbent. So the metal is Al.

Hence the answer is option (3).

50. Consider the following reaction and statements:



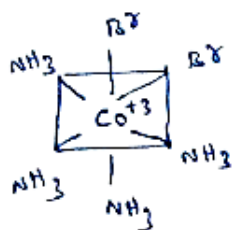
- (I) Two isomers are produced if the reactant complex ion is a cis-isomer.
(II) Two isomers are produced if the reactant complex ion is a trans-isomer.
(III) Only one isomer is produced if the reactant complex ion is a trans-isomer.
(IV) Only one isomer is produced if the reactant complex ion is a cis-isomer.

The correct statements are :

- (1) (I) and (II)
(2) (I) and (III)
(3) (III) and (IV)
(4) (II) and (IV)

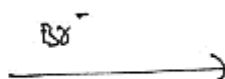
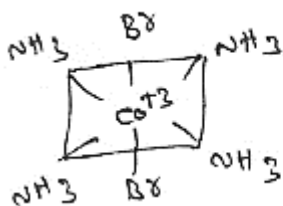
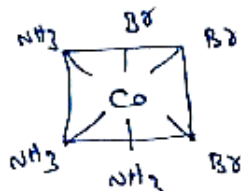
Solution:

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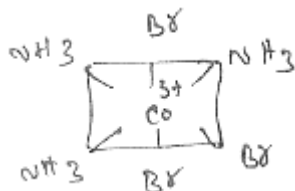
cis-isomer

$\xrightarrow{Br^-}$ 2 isomers can be produced



As all the NH_3 positions are identical

only one product can be formed.

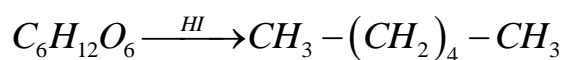


Hence the answer is option (2).

51. Glucose on prolonged heating with HI gives :

- (1) n-Hexane
- (2) 1-Hexane
- (3) Hexanoic acid
- (4) 6-iodohexanal

Solution:



Glucose n-hexane

Hence the answer is option (1).

52. The trans-alkenes are formed by the reduction of alkynes with :

- (1) $H_2 - Pd/C, BaSO_4$
- (2) $NaBH_4$
- (3) $Na/liq. NH_3$
- (4) $Sn-HCl$

Solution:


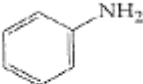
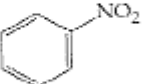
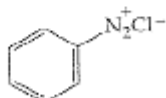
Trans alkenes are formed by the reaction of alkynes with $Na/liq. NH_3$
(birch Reduction)

Set: A

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Hence the answer is option (3).

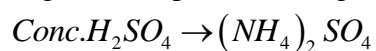
53. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

- (1) 
- (2) 
- (3) 
- (4) 

Solution:

Kjeldahl's method:-

Organic compounds nitrogen +



\downarrow alkali



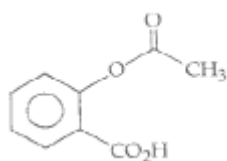
Nitro compounds, A_2O compounds & Nitrogen part of the aromatic ring will not give positive result for Kjeldahl's method.

Aniline is the best suitable to estimate nitrogen using Kjeldahl's method.

Hence the answer is option (2).

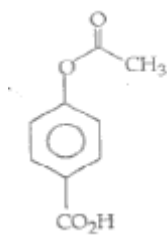
54. Phenol on treatment with CO_2 in the presence of $NaOH$ followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

(1)

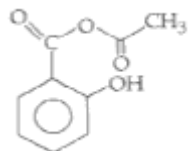


(2)

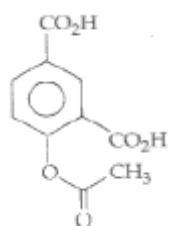
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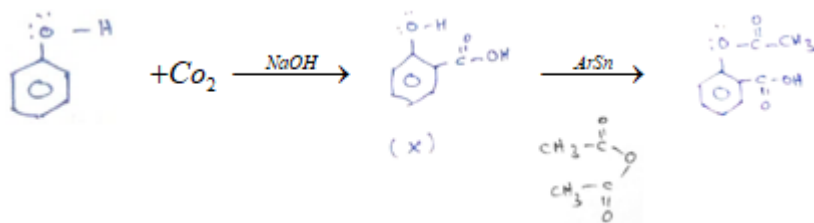
(3)



(4)



Solution:



Hence the answer is option (1).

55. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

	Base	Acid	End point
(1)	Weak	Strong	Colourless to pink
(2)	Strong	Strong	Pinkish red to yellow
(3)	Weak	Strong	Yellow to pinkish red
(4)	Strong	Strong	Pink to colourless

Solution:

When a weak base is titrated with strong acid with methyl orange as an indicator then at end point

The colour change will be yellow to pinkish red.

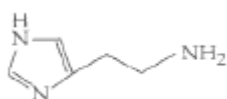
Hence the answer is option (3).

Set: A

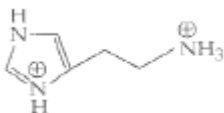
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56. The predominant form of histamine present in human blood is (pK_a , Histidine = 6.0)

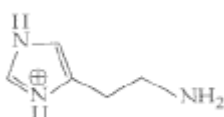
(1)



(2)



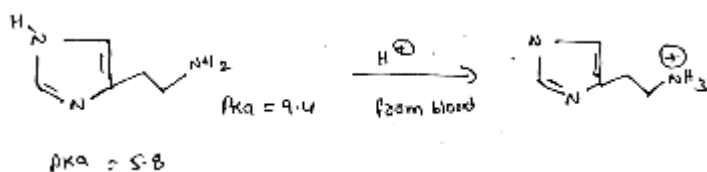
(3)



(4)



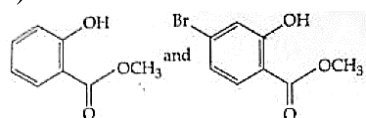
Solution:



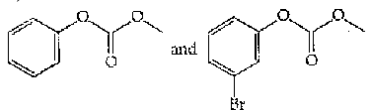
Hence the answer is option (4).

57. Phenol reacts with methyl chloroformate in the presence of $NaOH$ to form product A. A reacts with Br_2 to form product B. A and B are respectively :

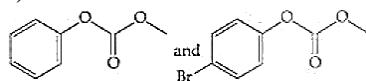
(1)



(2)



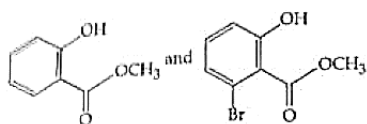
(3)



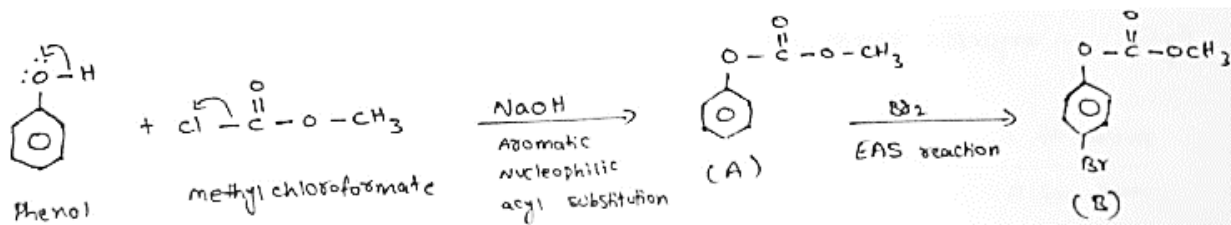
(4)

Set: A

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Solution:



Hence the answer is option (3).

58. The increasing order of basicity of the following compounds is :

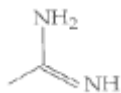
(a)



(b)



(c)



(d)



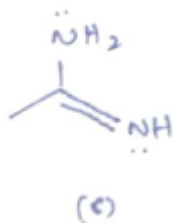
(1) (a) < (b) < (c) < (d)

(2) (b) < (a) < (c) < (d)

(3) (b) < (a) < (d) < (c)

(4) (d) < (b) < (a) < (c)

Solution:



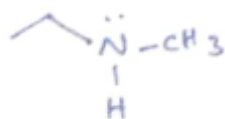
The lone pair on the nitrogen atom is in conjugation with the π bond hence it can involve in resonance.

during resonance the other nitrogen attains a negative charge.

Hence 'c' is a strong base.

Set: A

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(d)

The +I group of CH_3 and C_2H_5 makes 'd' more basic but less basic than 'c'



(a)

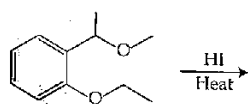
Due to -I of π bond, basic character decreases.



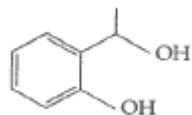
(b)

The nitrogen involves sp^2 hybridization which is highly electro negative have least basic. Hence the answer is option (4).

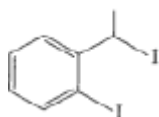
59. The major product formed in the following reaction is :



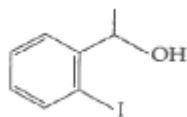
(1)



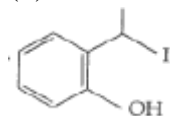
(2)



(3)

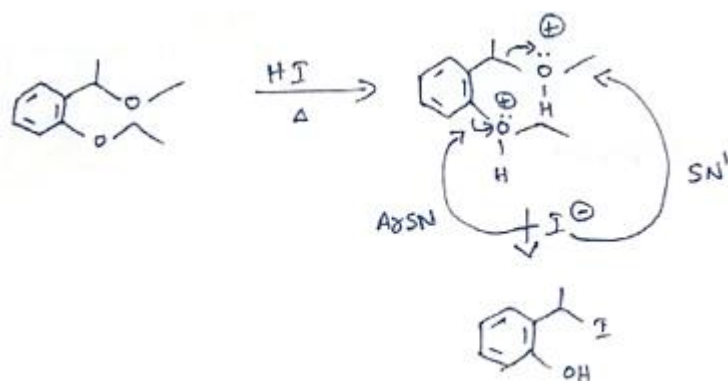


(4)



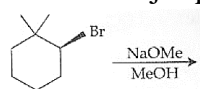
Solution:

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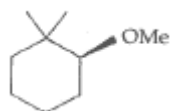


Hence the answer is option (4).

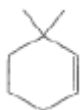
60. The major product of the following reaction is :



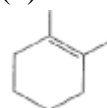
(1)



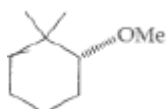
(2)



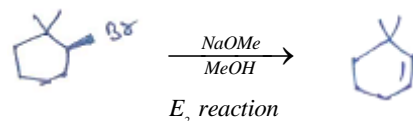
(3)



(4)



Solution:



Hence the answer is option (2).

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Mathematics

61. Two sets A and B are as under :

$$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\};$$

$$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}. \text{ Then :}$$

- (1) $B \subset A$
- (2) $A \subset B$
- (3) $A \cap B = \Phi$ (an empty set)
- (4) neither $A \subset B$ nor $B \subset A$

62. Let $S = \{x \in R : x \geq 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :

- (1) is an empty set.
- (2) contains exactly one element.
- (3) contains exactly two element.
- (4) contains exactly four elements.

Solution:

$$2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$$

Case-I $\sqrt{x} \geq 3$

$$\Rightarrow 2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0 \Rightarrow x - 4\sqrt{x} = 0 \Rightarrow x = 0, 6$$

$$\text{As } x \geq 9 \Rightarrow x = 16$$

$$\text{Case - II } \sqrt{x} < 3 \Rightarrow -2\sqrt{x} + 6 + x - 6\sqrt{x} + 6 = 0$$

$$(\sqrt{x} - 6)(\sqrt{x} - 2) = 0 \Rightarrow x = 36, 4$$

$$\text{As, } \sqrt{x} < 3 \Rightarrow x = 4$$

There are exactly two elements in the given set.

63. If $\alpha, \beta \in c$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :

- (1) -1
- (2) 0
- (3) 1
- (4) 2

Solution:

$$x^2 - x + 1 = 0$$

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$$x = \frac{1 \pm \sqrt{-3}}{2} = -\omega, -\omega^2$$

$$\Rightarrow \alpha = -\omega \text{ and } \beta = -\omega^2$$

$$\Rightarrow (-\omega)^{101} + (-\omega^2)^{107} = (\omega^{101} + \omega^{214}) = -(\omega^2 + \omega) = 1$$

64. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to :

- (1) (-4, -5)
- (2) (-4, 3)
- (3) (-4, 5)
- (4) (4, 5)

65. If the system of linear equations

$$x + ky + 3z = 0$$

$$3x + ky - 2z = 0$$

$$2x + 4y - 3z = 0$$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to:

- (1) -10
- (2) 10
- (3) -30
- (4) 30

66. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

- (1) at least 1000
- (2) less than 500
- (3) at least 500 but less than 750
- (4) at least 750 but less than 1000

67. The sum of the co-efficients of all odd degree terms in the expansion of

$$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, (x > 1) \text{ is:}$$

- (1) -1
- (2) 0
- (3) 1
- (4) 2

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68. Let $a_1, a_2, a_3, \dots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$, then m is equal to:

- (1) 66
- (2) 68
- (3) 34
- (4) 33

Solution:

$$a_1 + a_5 + a_9 = 416 \Rightarrow a + 24d = 32 \dots\dots\dots(i)$$

$$a_9 + a_{43} = 66 \Rightarrow a + 25d = 33 \dots\dots\dots(ii)$$

From (i) and (ii) $d = 1$ and $a = 8$

$$\text{Now, } a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$$

$$\Rightarrow \sum_{r=1}^{17} (8 + (r-1))^2 \Rightarrow \sum_{r=1}^{17} (7+r)^2 = 140m \Rightarrow m = 34$$

69. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$$

If $B - 2A = 100\lambda$, then λ is equal to:

- (1) 232
- (2) 248
- (3) 464
- (4) 496

Solution:

$$\begin{aligned} A &= 1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + \dots + A^2 + 2 \cdot 20^2 \\ &= (1^2 + 2 \cdot 2^2 + 3^2 + 4^2 + \dots + 20^2) + (2^2 + 4^2 + \dots + 20^2) \\ &= \frac{20 \times 21 \times 41}{6} + 4 \times \frac{10 \times 11 \times 21}{6} = 2870 + 1540 = 4410 = 2870 + 1540 = 4410 \end{aligned}$$

$$B = \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6} = 540 \times 41 + 41 \times 280 = 41 \times 820 = 33620$$

$$33620 - 8820 = 100\lambda$$

$$100\lambda = 24800$$

$$\lambda = 248$$

70. For each $t \in \mathbb{R}$, let $[t]$ be the greatest integer less than or equal to t . Then

$$\lim_{x \rightarrow 0^+} x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{15}{x} \right] \right)$$

(1) is equal to 0

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- (2) is equal to 50
- (3) is equal to 120
- (4) does not exist (in \mathbb{R})

71. Let $S = \{t \in \mathbb{R} : f(x) = |x - \pi| \cdot (e^{|x|} - 1) \sin|x| \text{ is not differentiable at } t\}$. Then the set S is equal to:

- (1) Φ (an empty set)
- (2) $\{0\}$
- (3) $\{\pi\}$
- (4) $\{0, \pi\}$

72. If the curve $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:

- (1) 6
- (2) $\frac{7}{2}$
- (3) 4
- (4) $\frac{9}{2}$

Solution:

$$2yy' = 6$$

$$y' = \frac{6}{2y} = \frac{3}{y_1}$$

$$18x_1 = \frac{18x_1}{2by_1} = \frac{-9x_1}{by_1} \Rightarrow -\frac{27x_1}{by_1^2} = -1 \Rightarrow b = \frac{27x_1}{y_1^2}$$

$$y_1^2 = 6x_1 \Rightarrow b = \frac{9}{2}$$

73. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in \mathbb{R} - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is :

- (1) 3
- (2) -3
- (3) $-2\sqrt{2}$
- (4) $\sqrt[3]{2}$

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74. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to :

(1) $\frac{1}{3(1 + \tan^3 x)} + c$

(2) $\frac{-1}{3(1 + \tan^3 x)} + c$

(3) $\frac{1}{1 + \cot^3 x} + c$

(4) $\frac{-1}{1 + \cot^3 x} + c$

Solution:

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

$$\int \frac{\tan^2 x \sec^6 x}{(\tan^5 x + \tan^2 x + \tan^3 x + 1)^2} dx$$

Put $\tan x = t \Rightarrow \sec^2 x = \frac{dt}{dx}$

$$\int \frac{t^2 (1+t^2)^2}{(t^3+1)^2 (t^2+1)^2} dt$$

$$t^3 + 1 = y$$

$$3t^2 = \frac{dy}{dt}$$

$$\frac{1}{3} \int \frac{dy}{y^2} = -\frac{1}{3(y)} + C = -\frac{1}{3(\tan^3 x + 1)} + C$$

75. The value of $\int_{\frac{\pi}{-2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is:

(1) $\frac{\pi}{8}$

(2) $\frac{\pi}{2}$

(3) 4π

(4) $\frac{\pi}{4}$

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76. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (g \circ f)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is :

- (1) $\frac{1}{2}(\sqrt{3}-1)$
 (2) $\frac{1}{2}(\sqrt{3}+1)$
 (3) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$
 (4) $\frac{1}{2}(\sqrt{2}-1)$

Solution:

$$g(x) = \cos x^2$$

$$f(x) = \sqrt{x}$$

$$g(f(x)) = \cos x$$

$$\text{given, } 18x^2 - 9\pi x + \pi^2 = 0 \Rightarrow (6x - \pi)(3x - \pi) = 0$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}$$

$$\text{Area} = \int_{\pi/6}^{\pi/3} \cos x \, dx = \frac{\sqrt{3}-1}{2}$$

77. Let $y=y(x)$ be the solution of the differential equation

$\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0, \Pi)$. If $y\left(\frac{\Pi}{2}\right) = 0$, then $y\left(\frac{\Pi}{6}\right)$ is equal to :

- (1) $\frac{4}{9\sqrt{3}}\Pi^2$
 (2) $\frac{-8}{9\sqrt{3}}\Pi^2$
 (3) $-\frac{8}{9}\Pi^2$
 (4) $-\frac{4}{9}\Pi^2$

Solution:

$$\frac{dy}{dx} + y \cot x = 4x \sec x \Rightarrow d(y \sin x) = 4x dx$$

Integrating both sides we get $y \sin x = 2x^2 + C$

$$\text{Also, } y\left(\frac{\pi}{2}\right) = 0 \Rightarrow C = -\frac{\pi^2}{2}$$

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$$\Rightarrow y \sin x = 2x^2 - \frac{\pi^2}{2} \Rightarrow y\left(\frac{\pi}{6}\right) = -\frac{8\pi^2}{9}$$

78. A straight line through a fixed point $(2, 3)$ intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

- (1) $3x + 2y = 6$
- (2) $2x + 3y = xy$
- (3) $3x + 2y = xy$
- (4) $3x + 2y = 6xy$

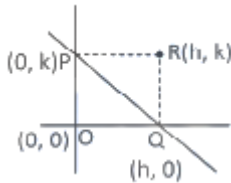
Solution:

Let $R = (h, k)$

$P = (0, k)$

$Q = (h, 0)$

Equation of line would be,



$$\frac{x}{h} + \frac{y}{k} = 1 \quad \dots\dots(i)$$

$$\Rightarrow \frac{2}{h} + \frac{3}{k} = 1$$

$$2k + 3h = hk$$

Locus of (h, k) is $2y + 3x = xy$

79. Let the orthocenter and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :

- (1) $\sqrt{10}$
- (2) $\sqrt[3]{10}$

(3) $\sqrt[3]{\frac{5}{2}}$

(4) $\frac{3\sqrt{5}}{2}$

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80. If the tangent at $(1, 7)$ to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :

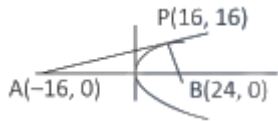
- (1) 195
- (2) 185
- (3) 85
- (4) 95

81. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B , respectively. If C is the centre of the circle through the points P , A and B and $\angle CPB = \theta$, then a value of $\tan \theta$ is :

- (1) $\frac{1}{2}$
- (2) 2
- (3) 3
- (4) $\frac{4}{3}$

Solution:

The equation of tangent at P



$$y - 16 = \frac{1}{2}(x - 16) \Rightarrow A = (-16, 0)$$

The normal is $y - 16 = -2(x - 16)$

$$B = (24, 0)$$

$$\text{Since } \angle APB = \frac{\pi}{2}$$

AB is the diameter

Centre of the circle $C = (4, 0)$

$$\text{Slope of } PB = -2 = m_1$$

$$\text{Slope of } CP = \frac{4}{3} = m_2 \Rightarrow \tan \theta = \left| \frac{m_2 - m_1}{1 + m_2 m_1} \right| = 2$$

82. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q . If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of ΔPTQ is :

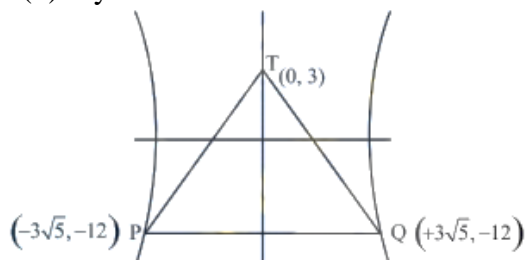
- (1) $45\sqrt{5}$
- (2) $54\sqrt{3}$
- (3) $60\sqrt{3}$
- (4) $36\sqrt{5}$

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Solution:

Equation of PQ,

$$4x(0) - 3y = 36$$



$$Y = -12$$

$$\text{Area of } \triangle TPQ = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$$

83. If L_1 is the line of intersection of the planes $2x - 2y + 3z = 0, x - y + z = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0, 3x - y + 2z = 0$ then the distance of the origin from the plane, containing the line L_1 and L_2 , is :

- (1) $\frac{1}{4\sqrt{2}}$
- (2) $\frac{1}{3\sqrt{2}}$
- (3) $\frac{1}{2\sqrt{2}}$
- (4) $\frac{1}{\sqrt{2}}$

84. The length of the projection of the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane, $x + y + z = 7$ is :

- (1) $\frac{2}{\sqrt{3}}$
- (2) $\frac{2}{3}$
- (3) $\frac{1}{3}$
- (4) $\sqrt{\frac{2}{3}}$

85. Let \vec{u} be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If \vec{u} is perpendicular to \vec{a} and $\vec{u} \cdot \vec{b} = 24$, then equal to :

- (1) 336
- (2) 315
- (3) 256
- (4) 84

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Solution:

$$\vec{u}(\vec{a} \times \vec{b}) = 0; \quad \vec{u} \cdot \vec{a} = 0 \text{ and } \vec{u} \cdot \vec{b} = 24$$

$$\text{Let } \vec{b} = (\vec{b} \cdot \vec{a})\vec{a} + (\vec{b} \cdot \vec{u})\vec{u}$$

$$|\vec{b}|^2 = (\vec{b} \cdot \vec{a})^2 + (\vec{b} \cdot \vec{u})^2$$

$$|\vec{b}|^2 = (\vec{b} \cdot \vec{a})^2 + \frac{(\vec{b} \cdot \vec{u})^2}{|\vec{u}|^2}$$

$$2 = \frac{2}{7} + \frac{(24)^2}{|\vec{u}|^2} \Rightarrow |\vec{u}|^2 = 336$$

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour is returned to the bag. If now a ball is drawn at random from the bag, then probability that this drawn ball is red, is:

(1) $\frac{3}{10}$

(2) $\frac{2}{5}$

(3) $\frac{1}{5}$

(4) $\frac{3}{4}$

87. If $\sum_{i=1}^9 (x_i - 5) = 9$ and $\sum_{i=1}^9 (x_i - 5)^2 = 45$, then the standard deviation of the 9 items

x_1, x_2, \dots, x_9 is:

(1) 9

(2) 4

(3) 2

(4) 3

88. If sum of all the solution of equation $8 \cos x \cdot \left(\cos \left(\frac{\pi}{6} + x \right) \cdot \cos \left(\frac{\pi}{6} - x \right) - \frac{1}{2} \right) = 1$ in

$[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{2}{3}$

(2) $\frac{13}{9}$

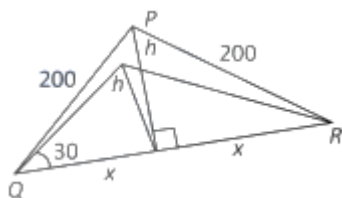
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- (3) $\frac{8}{9}$
 (4) $\frac{20}{9}$

89. PQR is a triangular park with $PQ=PR=200$ m. A.T.V. tower stands at the mid-point of QR. If the angle of elevation of the top of the tower at P,Q and R are respectively 45° , 30° and 30° then the height of tower (in m) is :

- (1) 100
 (2) 50
 (3) $100\sqrt{3}$
 (4) $50\sqrt{2}$

Solution:



$$\frac{h}{x} = \frac{1}{\sqrt{3}}$$

$$x = \sqrt{3}h$$

$$200 = 3h^2 + h^2$$

$$4h^2 = (200)^2$$

$$4h^2 = 40000$$

$$h = 100$$

90. The Boolean expression $\sim (p \vee q) \vee (\sim p \wedge q)$ is equivalent to:

- (1) $\sim p$
 (2) p
 (3) q
 (4) $\sim q$

Solution:

$$\sim (p \vee q) \vee (\sim p \wedge q)$$

P	q	$\sim (p \vee q)$	$\sim p \wedge q$
T	F	F	F
T	F	F	F
F	T	F	T
F	F	T	F

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BOOKLET CODE : A

Q.No.	Answer
1.	(3)
2.	(2)
3.	(2)
4.	(3)
5.	(2)
6.	(4)
7.	(1)
8.	(3)
9.	(3)
10.	(3)
11.	(1)
12.	(2)
13.	(1)
14.	(2)
15.	(1)
16.	(2)
17.	(2)
18.	(2)
19.	(3)
20.	(1)
21.	(3)
22.	(3)
23.	(1)
24.	(1)
25.	(4)
26.	(1)
27.	(3)
28.	(3)
29.	(2)
30.	(3)

Q.No.	Answer
31.	(4)
32.	(3)
33.	(4)
34.	(1)
35.	(4)
36.	(4)
37.	(2)
38.	(3)
39.	(1)
40.	(3)
41.	(3)
42.	(3)
43.	(4)*
44.	(3)
45.	(2)
46.	(3)
47.	(3)
48.	(4)
49.	(3)
50.	(2)
51.	(1)
52.	(3)
53.	(2)
54.	(1)
55.	(3)
56.	(4)
57.	(3)
58.	(3)
59.	(4)
60.	(2)

Q.No.	Answer
61.	(2)
62.	(3)
63.	(3)
64.	(3)
65.	(2)
66.	(1)
67.	(4)
68.	(3)
69.	(2)
70.	(3)
71.	(1)
72.	(4)
73.	(4)
74.	(2)
75.	(4)
76.	(1)
77.	(3)
78.	(3)
79.	(3)
80.	(4)
81.	(2)
82.	(1)
83.	(2)
84.	(4)
85.	(1)
86.	(2)
87.	(3)
88.	(2)
89.	(1)
90.	(1)