Series EF1GH/C

SET~1

प्रश्न–पत्र कोड ***Q.P. Code*** **65/C/1**

रोल नं.

Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

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निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।***

*Please check that this question paper contains **23** printed pages.*

(ii) ***प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।***

Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) ***कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।***

*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***

Please write down the serial number of the question in the answer-book before attempting it.

(v) ***इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।***

15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में* ***38*** *प्रश्न हैं ।* ***सभी*** *प्रश्न* ***अनिवार्य*** *हैं ।*

(ii) *यह प्रश्न-पत्र* ***पाँच*** *खण्डों में विभाजित है –* ***क, ख, ग, घ*** *एवं* ***ङ*** *।*

(iii) ***खण्ड क*** *में प्रश्न संख्या* ***1*** *से* ***18*** *तक बहुविकल्पीय तथा प्रश्न संख्या* ***19*** *एवं* ***20*** *अभिकथन एवं तर्क आधारित* ***एक-एक*** *अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख*** *में प्रश्न संख्या* ***21*** *से* ***25*** *तक अति लघु-उत्तरीय (VSA) प्रकार के* ***दो-दो*** *अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग*** *में प्रश्न संख्या* ***26*** *से* ***31*** *तक लघु-उत्तरीय (SA) प्रकार के* ***तीन-तीन*** *अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ*** *में प्रश्न संख्या* ***32*** *से* ***35*** *तक दीर्घ-उत्तरीय (LA) प्रकार के* ***पाँच-पाँच*** *अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ*** *में प्रश्न संख्या* ***36*** *से* ***38*** *प्रकरण अध्ययन आधारित* ***चार-चार*** *अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग* ***वर्जित*** *है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. यदि A, कोटि 3 का एक वर्ग आव्यूह है और $|A| = 6$ है, तो $|\text{adj } A|$ का मान है :

(a) 6 (b) 36

(c) 27 (d) 216

2. $\int_0^{\pi/6} \sin 3x \, dx$ का मान है :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

2. The value of $\int_0^{\pi/6} \sin 3x \, dx$ is :

(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$

3. यदि $\vec{a}$, $\vec{b}$ और $(\vec{a} + \vec{b})$ सभी मात्रक सदिश हैं और $\vec{a}$ तथा $\vec{b}$ के बीच का कोण θ है, तो θ का मान होगा :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

4. सदिश $\hat{i}$ का सदिश $\hat{i} + \hat{j} + 2\hat{k}$ पर प्रक्षेप है :

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$

5. एक परिवार में 2 बच्चे हैं और बड़ा बच्चा एक लड़की है । दोनों बच्चों के लड़की होने की प्रायिकता है :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

6. रेखा $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ के समांतर और बिंदु $(2, -4, 5)$ से गुज़रने वाली रेखा का सदिश समीकरण है :

(a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$

(c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$

7. x के किस मान के लिए, सारणिक $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ और $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ समान हैं ?

(a) ± 3 (b) -3 (c) ± 2 (d) 2

8. आव्यूह $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ में दूसरी पंक्ति और तीसरे स्तम्भ में स्थित अवयव के सहखंड का मान है :

(a) 5 (b) -5 (c) -11 (d) 11

3. If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

4. The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$

5. A family has 2 children and the elder child is a girl. The probability that both children are girls is :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

6. The vector equation of a line which passes through the point $(2, -4, 5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is :

(a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$

(c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$

7. For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ?

(a) ± 3 (b) -3 (c) ± 2 (d) 2

8. The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is :

(a) 5 (b) -5 (c) -11 (d) 11

9. अवकल समीकरण $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ की कोटि और घात में अंतर है :

(a) 1 (b) 2 (c) -1 (d) 0

10. यदि आव्यूह $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ और $A^2 = kA$ है, तो k का मान होगा :

(a) 1 (b) -2 (c) 2 (d) -1

11. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ बराबर है :

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

12. अवकल समीकरण $(3x^2 + y)\frac{dx}{dy} = x$ का समाकलन गुणक है :

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

13. अर्ध-तल $2x + y - 4 \leq 0$ में स्थित बिंदु है :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

14. यदि $(\cos x)^y = (\cos y)^x$ है, तो $\frac{dy}{dx}$ बराबर है :

(a) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$

(b) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$

(c) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$

(d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$

9. The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is :

(a) 1 (b) 2 (c) -1 (d) 0

10. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is :

(a) 1 (b) -2 (c) 2 (d) -1

11. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to

(a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$

(c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$

12. The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

13. The point which lies in the half-plane $2x + y - 4 \leq 0$ is :

(a) (0, 8) (b) (1, 1)

(c) (5, 5) (d) (2, 2)

14. If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$

(b) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$

(c) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$

(d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$

15. यह दिया गया है कि $X\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix} = \begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$ है । तो आव्यूह X है :

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\ 1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$

16. यदि ABCD एक समांतर चतुर्भुज है और AC तथा BD इसके विकर्ण हैं, तो $\vec{AC} + \vec{BD}$ है :

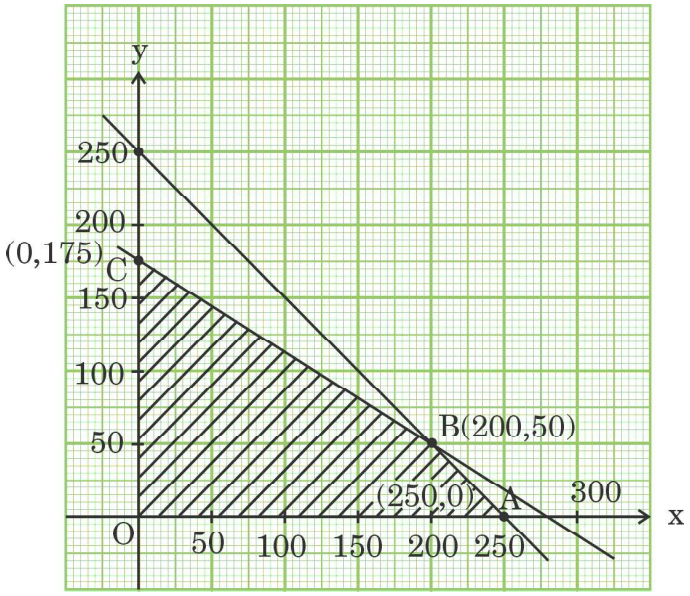
(a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$

17. यदि $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$ है, तो निम्नलिखित में से कौन-सा सही है ?

(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

18. एक LPP के परिबद्ध सुसंगत क्षेत्र के कोणीय बिंदु O(0, 0), A(250, 0), B(200, 50) और C(0, 175) हैं । यदि उद्देश्य फलन $Z = 2ax + by$ का अधिकतम मान बिंदुओं A(250, 0) और B(200, 50) पर है, तो a और b के बीच का संबंध होगा :



(a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$

15. It is given that $X\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Then matrix X is :

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

16. If ABCD is a parallelogram and AC and BD are its diagonals, then $\overrightarrow{AC} + \overrightarrow{BD}$ is :

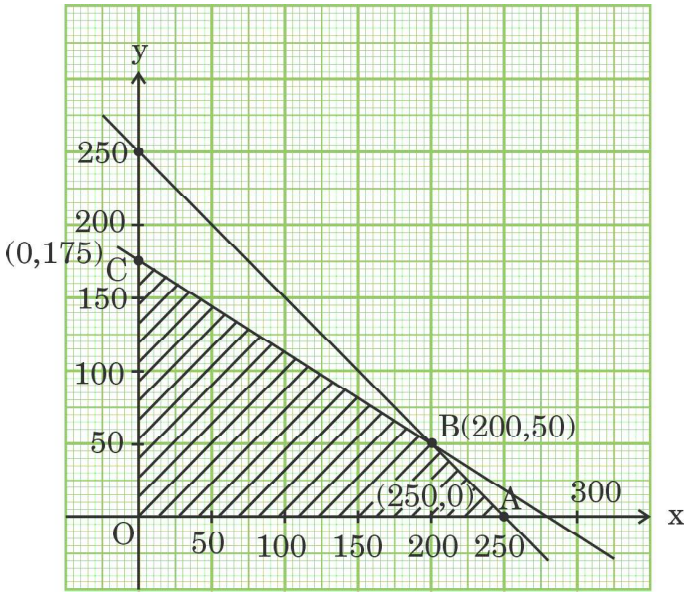
(a) $2\overrightarrow{DA}$ (b) $2\overrightarrow{AB}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{BD}$

17. If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ?

(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

18. The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function $Z = 2ax + by$ occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :



(a) $2a = b$ (b) $2a = 3b$ (c) $a = b$ (d) $a = 2b$

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं और प्रत्येक प्रश्न का 1 अंक है । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (a), (b), (c) और (d) में से चुनकर दीजिए ।*

(a) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(b) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(c) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(d) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* $\cot^{-1}(\sqrt{3})$ का मुख्य मान $\frac{\pi}{6}$ है ।

तर्क (R) : $\cot^{-1} x$ का प्रांत $\mathbb{R} - \{-1, 1\}$ है ।

20. *अभिकथन (A) :* शीर्षों A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) और D(5, 4, 3) से बना चतुर्भुज एक समचतुर्भुज है ।

तर्क (R) : ABCD एक समचतुर्भुज है, यदि AB = BC = CD = DA, $AC \neq BD$ है ।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. यदि तीन शून्येतर सदिश $\vec{a}$, $\vec{b}$ और $\vec{c}$ ऐसे हैं कि $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ और $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ है, तो दर्शाइए कि $\vec{b} = \vec{c}$ ।

22. (क) सरल कीजिए :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

अथवा

(ख) सिद्ध कीजिए कि $f(x) = [x]$ द्वारा प्रदत्त महत्तम पूर्णांक फलन $f : \mathbb{R} \to \mathbb{R}$ न तो एकैकी है और न ही आच्छादक है ।

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

20. *Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus.

Reason (R) : ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If three non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.

22. (a) Simplify :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$$

OR

(b) Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

23. फलन f इस प्रकार परिभाषित है

$$f(x) = \begin{cases} 2x + 2, & \text{यदि} \quad x < 2 \\ k, & \text{यदि} \quad x = 2 \\ 3x, & \text{यदि} \quad x > 2 \end{cases}$$

k का वह मान ज्ञात कीजिए, जिसके लिए फलन f, $x = 2$ पर संतत है ।

24. वह अंतराल ज्ञात कीजिए जिसमें फलन $f(x) = x^4 - 4x^3 + 4x^2 + 15$, निरंतर वर्धमान है ।

25. (क) यदि $\vec{a}$, $\vec{b}$ और $\vec{c}$ तीन सदिश इस प्रकार हैं कि $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ और $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ है, तो $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ का मान ज्ञात कीजिए ।

अथवा

(ख) यदि एक रेखा x-अक्ष, y-अक्ष और z-अक्ष के साथ क्रमशः α, β और γ कोण बनाती है, तो सिद्ध कीजिए कि $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$ है ।

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. (क) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_1^3 (|x - 1| + |x - 2|) \, dx$$

23. Function f is defined as

$$f(x) = \begin{cases} 2x + 2, & \text{if} \quad x < 2 \\ k, & \text{if} \quad x = 2 \\ 3x, & \text{if} \quad x > 2 \end{cases}$$

Find the value of k for which the function f is continuous at x = 2.

24. Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing.

25. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

(b) If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Evaluate :

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x-1| + |x-2|) \, dx$$

27. (क) अवकल समीकरण $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि जब $x = 0$ है, तो $y = 1$ है ।

अथवा

(ख) अवकल समीकरण $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है कि जब $x = 1$ है, तो $y = 0$ है ।

28. (क) दो थैलों में से थैले A में 2 सफ़ेद और 3 लाल गेंदें हैं और थैले B में 4 सफ़ेद और 5 लाल गेंदें हैं । यादृच्छया एक गेंद को एक थैले में से निकाला गया और पाया गया कि यह लाल है । प्रायिकता ज्ञात कीजिए कि इसे थैले B में से निकाला गया था ।

अथवा

(ख) 50 व्यक्तियों के समूह में से 20 सदैव सच बोलते हैं । इस समूह में से यादृच्छया 2 व्यक्तियों को चुना गया (बिना प्रतिस्थापना के) । चुने गए उन व्यक्तियों की संख्या का प्रायिकता बंटन ज्ञात कीजिए जो सदैव सच बोलते हैं ।

29. ज्ञात कीजिए :

$$\int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$$

30. निम्नलिखित रैखिक प्रोग्रामन समस्या को आलेखीय विधि से हल कीजिए :

निम्न व्यवरोधों के अंतर्गत,

$z = 3x + 8y$ का न्यूनतमीकरण कीजिए :

$$3x + 4y \geq 8$$
$$5x + 2y \geq 11$$
$$x \geq 0, y \geq 0$$

31. ज्ञात कीजिए :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that y = 1 when x = 0.

OR

(b) Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$, given that y = 0 when x = 1.

28. (a) Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

OR

(b) Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

29. Find :

$$\int \frac{\cos\theta}{\sqrt{3 - 3\sin\theta - \cos^2\theta}}\, d\theta$$

30. Solve the following Linear Programming Problem graphically :

Minimise z = 3x + 8y

subject to the constraints

$$3x + 4y \geq 8$$
$$5x + 2y \geq 11$$
$$x \geq 0, y \geq 0$$

31. Find :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं ।

32. यदि आव्यूह $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए, अत: निम्नलिखित रैखिक समीकरण निकाय को हल कीजिए :

$$3x + 2y + z = 2000$$
$$4x + y + 3z = 2500$$
$$x + y + z = 900$$

33. (क) दर्शाइए कि रेखाएँ $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ और $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ प्रतिच्छेदी रेखाएँ हैं । इनका प्रतिच्छेदन बिन्दु भी ज्ञात कीजिए ।

अथवा

(ख) रेखा युग्मों $\frac{x-1}{2} = \frac{y+1}{3} = z$ और $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$ के बीच की न्यूनतम दूरी ज्ञात कीजिए ।

34. त्रिभुज ABC का क्षेत्रफल समाकलन विधि के प्रयोग से ज्ञात कीजिए जो कि रेखाओं जिनके समीकरण $5x - 2y - 10 = 0$, $x - y - 9 = 0$ और $3x - 4y - 6 = 0$ हैं, से घिरा हुआ है ।

35. (क) दर्शाइए कि वास्तविक संख्याओं के समुच्चय $\mathbb{R}$ में

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

द्वारा परिभाषित संबंध S न तो स्वतुल्य है, न सममित है और न ही संक्रामक है ।

अथवा

(ख) माना कि समुच्चय $A = \{1, 2, 3, 4, 5, 6, 7\}$ में संबंध R इस प्रकार परिभाषित है

$R = \{(a, b) : a$ और b दोनों या तो विषम हैं या सम हैं$\}$ दर्शाइए कि R एक तुल्यता संबंध है । अत:, तुल्यता वर्ग [1] के अवयव ज्ञात कीजिए ।

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. If matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$3x + 2y + z = 2000$$
$$4x + y + 3z = 2500$$
$$x + y + z = 900$$

33. (a) Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

(b) Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$.

34. Find the area of the triangle ABC bounded by the lines represented by the equations $5x - 2y - 10 = 0$, $x - y - 9 = 0$ and $3x - 4y - 6 = 0$, using integration method.

35. (a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by

R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं ।

प्रकरण अध्ययन – 1

36. एक समूह क्रियाकलाप की कक्षा में 10 विद्यार्थी हैं जिनकी आयु 16, 17, 15, 14, 19, 17, 16, 19, 16 और 15 वर्ष हैं । एक विद्यार्थी को यादृच्छया इस प्रकार चुना गया कि प्रत्येक विद्यार्थी के चुने जाने की संभावना समान है और चुने गए विद्यार्थी की आयु को लिखा गया ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) प्रायिकता ज्ञात कीजिए कि चुने गए विद्यार्थी की आयु एक भाज्य संख्या है । *1*

(ii) माना X चुने हुए विद्यार्थी की आयु है, तो X का क्या मान हो सकता है ? *1*

(iii) (क) यादृच्छया चर X का प्रायिकता बंटन ज्ञात कीजिए तथा माध्य आयु ज्ञात कीजिए । *2*

अथवा

(iii) (ख) एक यादृच्छया चुने गए विद्यार्थी की आयु 15 वर्ष से अधिक पाई गई । प्रायिकता ज्ञात कीजिए कि उसकी आयु एक अभाज्य संख्या है । *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

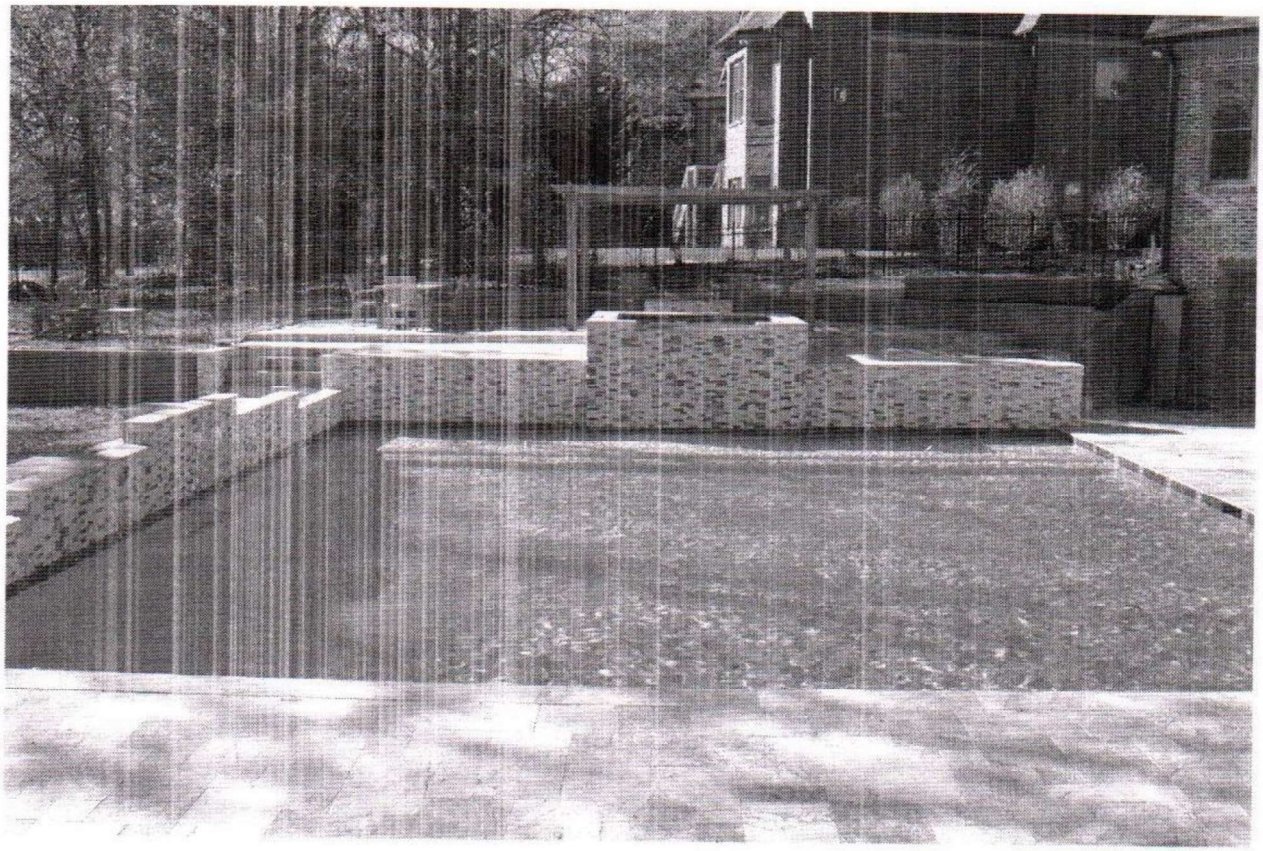
(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

प्रकरण अध्ययन – 2

37. एक हाउसिंग सोसाइटी अपने निवासियों के लिए तैराकी हेतु एक पूल (तालाब) बनाना चाहती है । इसके लिए उन्हें एक वर्गाकार भूमि खरीदनी है और इस गहराई तक खोदना है कि इस पूल की क्षमता 250 घन मीटर हो जाए । भूमि की कीमत ₹ 500 प्रति वर्ग मीटर है । खोदने की कीमत में गहराई की अधिकता के अनुसार वृद्धि होती जाती है तथा पूरे पूल की लागत ₹ 4000 (गहराई)^2 है ।



मान लीजिए कि वर्गाकार प्लॉट की भुजा x मीटर और गहराई h मीटर है ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) लागत फलन C(h) को h के पदों में लिखिए । *1*

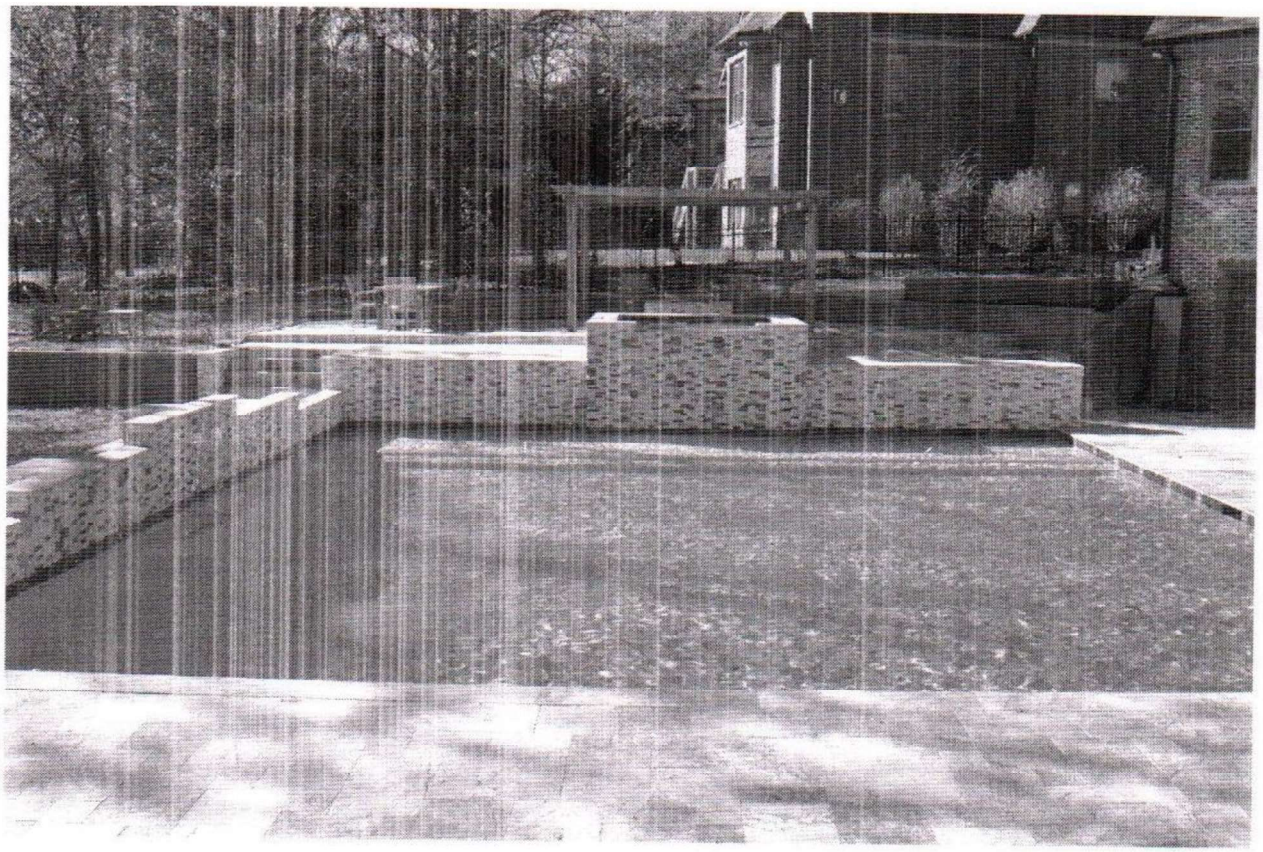
(ii) क्रांतिक बिंदु ज्ञात कीजिए । *1*

(iii) (क) द्वितीय अवकलज परीक्षण द्वारा h का वह मान ज्ञात कीजिए, जिसके लिए पूल बनाने की लागत न्यूनतम हो । पूल बनाने की न्यूनतम लागत क्या है ? *2*

अथवा

Case Study – 2

37. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.



Suppose the side of the square plot is x metres and depth is h metres.

On the basis of the above information, answer the following questions :

(i) Write cost C(h) as a function in terms of h. *1*

(ii) Find critical point. *1*

(iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2*

OR

(iii) (ख) प्रथम अवकलज परीक्षण से पूल की ऐसी गहराई ज्ञात कीजिए कि पूल बनाने की लागत न्यूनतम हो । न्यूनतम लागत के लिए x और h के बीच का संबंध भी ज्ञात कीजिए । *2*

प्रकरण अध्ययन – 3

38. एक कृषि संस्थान में, वैज्ञानिक बीजों की किस्मों को अलग-अलग वातावरणों में उगाने का प्रयोग करते हैं जिससे कि स्वस्थ पौधे उगें और अधिक उपज प्राप्त हो ।

एक वैज्ञानिक ने अवलोकन किया कि एक विशेष बीज अंकुरित होने के बाद बहुत तेज़ी से बढ़ रहा है । उसने बीज के अंकुरण के बाद से ही पौधे की वृद्धि को रिकॉर्ड किया था और उसने कहा कि इस वृद्धि को फलन $f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2,\ 0 \le x \le 10$ से परिभाषित किया जा सकता है, जहाँ x दिनों की वह संख्या है जिनमें पौधा सूर्य के प्रकाश से उजागर था ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) इस फलन f(x) के क्रांतिक बिंदु कौन-से हैं ? *2*

(ii) द्वितीय अवकलज परीक्षण का प्रयोग करके, फलन का न्यूनतम मान ज्ञात कीजिए । *2*

(iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*

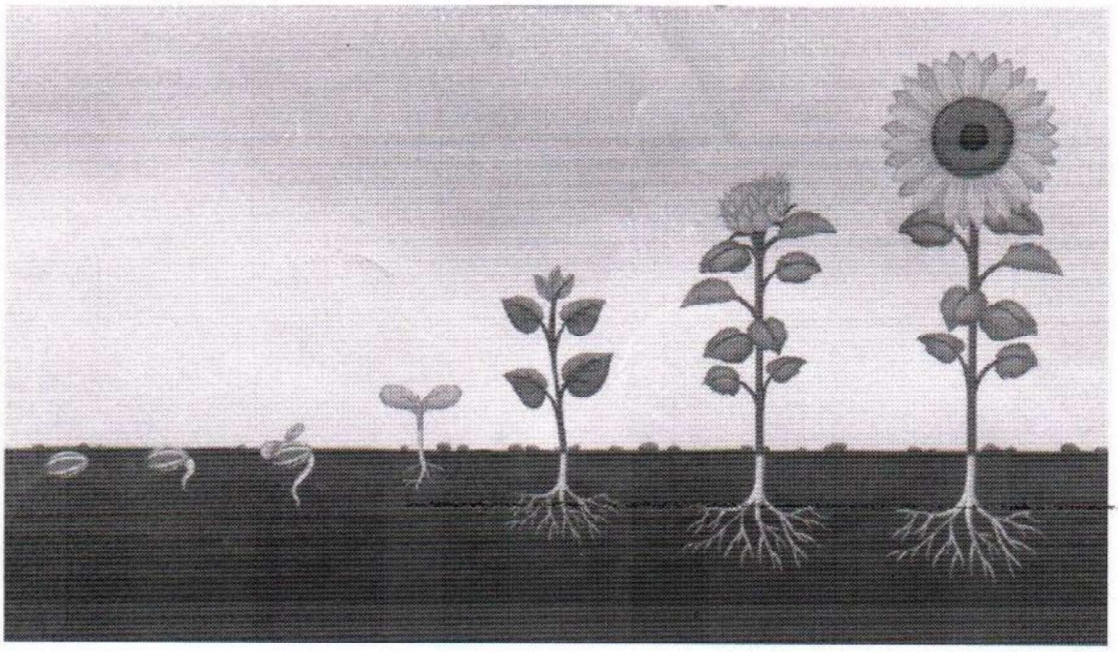
Case Study – 3

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \quad 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? *2*

(ii) Using second derivative test, find the minimum value of the function. *2*

Marking Scheme
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Sr. Secondary School Supplementary Examination, July- 2023
MATHEMATICS PAPER CODE 65/C/1

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**

11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

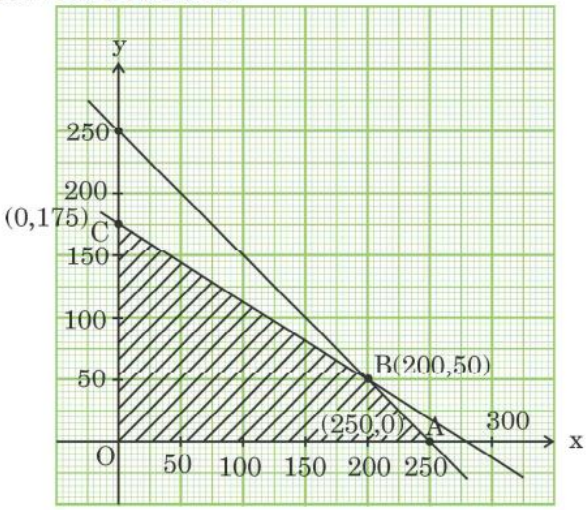
MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/C/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	If A is a square matrix of order 3 and $\|A\| = 6$, then the value of $\|\text{adj } A\|$ is : (a) 6 (b) 36 (c) 27 (d) 216	
Sol.	**(b) 36**	**1**
2.	The value of $\int_0^{\pi/6} \sin 3x \, dx$ is : (a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$	
Sol.	**(d) $\frac{1}{3}$**	**1**
3.	If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is : (a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$	
Sol.	**(a) $\frac{2\pi}{3}$**	**1**

4.	The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is : (a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$	
Sol.	(a) $\frac{1}{\sqrt{6}}$	**1**
5.	A family has 2 children and the elder child is a girl. The probability that both children are girls is : (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$	
Sol.	(c) $\frac{1}{2}$	**1**
6.	The vector equation of a line which passes through the point $(2, -4, 5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is : (a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$ (c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$	
Sol.	(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$	**1**
7.	For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ? (a) ± 3 (b) -3 (c) ± 2 (d) 2	
Sol.	(c) ± 2	**1**
8.	The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is : (a) 5 (b) -5 (c) -11 (d) 11	
Sol.	(b) -5	**1**

9.	The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is : (a) 1 (b) 2 (c) -1 (d) 0	
Sol.	**(d) 0**	**1**
10.	If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is : (a) 1 (b) -2 (c) 2 (d) -1	
Sol.	**(c) 2**	**1**
11.	$\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to (a) $\tan x - \cot x + C$ (b) $-\cot x - \tan x + C$ (c) $\cot x + \tan x + C$ (d) $\tan x - \cot x - C$	
Sol.	**(b) $-\cot x - \tan x + C$**	**1**
12.	The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is (a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$	
Sol.	(a) $\frac{1}{x}$	**1**
13.	The point which lies in the half-plane $2x + y - 4 \leq 0$ is : (a) (0, 8) (b) (1, 1) (c) (5, 5) (d) (2, 2)	
Sol.	(b) (1, 1)	**1**
14.	If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :	

	(a) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$ (b) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$ (c) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$ (d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$	
Sol.	(d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$	**1**
15.	It is given that $X\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix} = \begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$. Then matrix X is : (a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & -1\\ 1 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$	
Sol.	(c) $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$	**1**
16.	If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is : (a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$	
Sol.	(c) $2\vec{BC}$	**1**
17.	If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ? (a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$	
Sol.	(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$	**1**

18.	The corner points of the bounded feasible region of an LPP are O(0, 0), A(250, 0), B(200, 50) and C(0, 175). If the maximum value of the objective function Z = 2ax + by occurs at the points A(250, 0) and B(200, 50), then the relation between a and b is :  (a) 2a = b (b) 2a = 3b (c) a = b (d) a = 2b	
Sol.	(a) 2a = b	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true and Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$. *Reason (R) :* Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.	
Sol.	(c) Assertion (A) is true, but Reason (R) is false.	**1**

20.	*Assertion (A) :* Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus. *Reason (R) :* ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21.	If three non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.	
Sol.	$\vec{a}.\vec{b} = \vec{a}.\vec{c} \Rightarrow \vec{a}.\left(\vec{b} - \vec{c}\right) = 0$ $\Rightarrow \vec{a} = 0 \text{ or } \vec{b} - \vec{c} = 0 \text{ or } \vec{a} \perp \left(\vec{b} - \vec{c}\right)$ $\text{as } \vec{a} \neq 0 \Rightarrow \vec{b} - \vec{c} = 0 \text{ or } \vec{a} \perp \left(\vec{b} - \vec{c}\right) \quad ...(1)$ $\text{Again, } \vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times \left(\vec{b} - \vec{c}\right) = 0$ $\Rightarrow \vec{a} = 0 \text{ or } \vec{b} - \vec{c} = 0 \text{ or } \vec{a} \parallel \left(\vec{b} - \vec{c}\right).$ $\text{as } \vec{a} \neq 0 \Rightarrow \vec{b} - \vec{c} = 0 \text{ or } \vec{a} \parallel \left(\vec{b} - \vec{c}\right) \quad ...(2)$ $\text{from } (1) \text{ and } (2), \vec{b} = \vec{c} \quad (\because \vec{a} \text{ can't be parallel and perpendicular to } \left(\vec{b} - \vec{c}\right) \text{ simultaneously.})$	**1** ½ ½
22(a).	Simplify : $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$	

Sol.	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$ $=\tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$ $=\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$ $=\frac{\pi}{4}+\frac{x}{2}$	½ ½ ½ ½
	OR	
22(b).	Prove that the greatest integer function $f:\mathbb{R}\to\mathbb{R}$, given by f(x) = [x], is neither one-one nor onto.	
Sol.	For not one-one: $1.1,1.2\in R$ (domain) now, $1.1\neq1.2$ but $f(1.1)=f(1.2)=1\Rightarrow f$ is not one-one. For not onto : Let $\frac{1}{2}\in R$ (co-domain), but $[x]=\frac{1}{2}$ is not possible for x in domain. so, f is not onto.	**1** **1**
23.	Function f is defined as $f(x)=\begin{cases}2x+2, & \text{if } x<2\\ k, & \text{if } x=2\\ 3x, & \text{if } x>2\end{cases}$ Find the value of k for which the function f is continuous at x = 2.	
Sol.	As f is continuous at $x=2\Rightarrow\lim_{x\to2^+}f(x)=\lim_{x\to2^-}f(x)=f(2)$ $\lim_{x\to2^+}3x=\lim_{x\to2^-}(2x+2)=k$ $\Rightarrow k=6$	**1** **1**

24.	Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$, is strictly increasing.	
Sol.	$f'(x) = 4x^3 - 12x^2 + 8x = 4x(x-1)(x-2)$ $f'(x) = 0$ gives $x = 0, 1, 2$ for strictly increasing, $f'(x) > 0$ $x \in (0,1) \cup (2, \infty)$	**1** **1**
25(a).	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\lvert\vec{a}\rvert = 7$, $\lvert\vec{b}\rvert = 24$, $\lvert\vec{c}\rvert = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.	
Sol.	$\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = (\vec{0})^2$ $\Rightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow 49 + 576 + 625 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -625$	**1** **1**
	OR	
25(b).	If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.	
Sol.	d.c. are $\cos\alpha, \cos\beta, \cos\gamma$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$ $\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$	**1** **1**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	

26(a).	Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$	
Sol.	$I = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx \quad ...(1)$ using $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}\, dx$ $\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\cos x \sin x}{\cos^4 x + \sin^4 x}\, dx \quad ...(2)$ adding (1) and (2) $2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\, dx = \pi \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\, dx$ $\Rightarrow I = \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{\left(\tan^2 x\right)^2 + 1}\, dx \quad (\because \text{dividing by } \cos^4 x)$ Putting $\tan^2 x = t$ gives $I = \frac{\pi}{4}\int_0^1 \frac{1}{t^2+1}\, dt$ $\Rightarrow I = \frac{\pi^2}{16}$	½ ½ **1** ½ ½
	OR	
26(b).	Evaluate : $\int_1^3 (\lvert x-1 \rvert + \lvert x-2 \rvert)\, dx$	

Sol.	$I=\int_1^3 \left(\lvert x-1\rvert+\lvert x-2\rvert\right)dx$ $=\int_1^2\left[(x-1)-(x-2)\right]dx+\int_2^3\left[(x-1)+(x-2)\right]dx$ $=\int_1^2 1\,dx+\int_2^3(2x-3)\,dx$ $=\left[x\right]_1^2+\left[x^2-3x\right]_2^3$ $=1+2=3$	**1** ½ ½ **1**
27(a).	Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that y = 1 when x = 0.	
Sol.	$\frac{dy}{dx}=\frac{xy}{x^2+y^2}\quad ...(1)$ Put $\frac{y}{x}=v$ i.e. $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ Equation (1) gives $v+x\frac{dv}{dx}=\frac{v}{1+v^2}$ $\Rightarrow x\frac{dv}{dx}=-\frac{v^3}{1+v^2}$ $\Rightarrow\int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$ $\Rightarrow\frac{-1}{2v^2}+\log\lvert v\rvert=-\log\lvert x\rvert+\log c$ putting $v=\frac{y}{x}$ and simplifying gives $-\frac{x^2}{2y^2}=\log\left\lvert\frac{c}{y}\right\rvert$ now, $x=0, y=1$ gives $c=1$ required solution is : $\frac{x^2}{2y^2}=\log\lvert y\rvert$	½ **1** ½ ½ ½
	OR	

27(b).	Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given that y = 0 when x = 1.	
Sol.	Given diff. eqn. can be written as $\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{\left(1+x^2\right)^2}$ I.F. $=e^{\int\frac{2x}{1+x^2}dx}=1+x^2$ solution is given by: $y.\left(1+x^2\right)=\int\frac{1}{1+x^2}dx$ $\Rightarrow y.\left(1+x^2\right)=\tan^{-1}x+C$ Now $x=1, y=0$ gives $C=-\frac{\pi}{4}$ Reqired solution : $y.\left(1+x^2\right)=\tan^{-1}x-\frac{\pi}{4}$	½ ½ ½ ½ ½ ½
28(a).	Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.	
Sol.	Let E_1 : event of choosing bag A, E_2 : event of choosing bag B, A : red ball is found here, $P(E_1)=P(E_2)=\frac{1}{2}; P(A\mid E_1)=\frac{3}{5}, P(A\mid E_2)=\frac{5}{9}$ $P(E_2\mid A)=\frac{P(E_2)P(A\mid E_2)}{P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)}$ $=\frac{\frac{5}{9}\times\frac{1}{2}}{\frac{3}{5}\times\frac{1}{2}+\frac{5}{9}\times\frac{1}{2}}=\frac{25}{52}$	½ **1** **1+ ½**
	OR	

28(b). Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of number of selected persons who always speak the truth.

Sol. Let X be the random variable representing the number of persons who speak truth. ½

X can takes the values 0, 1 and 2. ½

$P(\text{speaking truth}) = \frac{20}{50}, P(\text{not speaking truth}) = \frac{30}{50}$

$P(X=0) = \frac{30}{50} \times \frac{29}{49} = \frac{87}{245}$

$P(X=1) = 2 \times \frac{20}{50} \times \frac{30}{49} = \frac{120}{245}$ **1½**

$P(X=2) = \frac{20}{50} \times \frac{19}{49} = \frac{38}{245}$

Probability Distribution Table is given by:

X	**0**	**1**	**2**
$P(X)$	$\frac{87}{245}$	$\frac{120}{245}$	$\frac{38}{245}$

½

29. Find :

$$\int \frac{\cos\theta}{\sqrt{3-3\sin\theta-\cos^2\theta}}\, d\theta$$

Sol.

$$I = \int \frac{\cos\theta}{\sqrt{3-3\sin\theta-\cos^2\theta}}\, d\theta$$

$$= \int \frac{\cos\theta}{\sqrt{\sin^2\theta-3\sin\theta+2}}\, d\theta$$ ½

Put $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$ ½

$$I = \int \frac{dt}{\sqrt{t^2-3t+2}} = \int \frac{dt}{\sqrt{\left(t-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$ ½

$$= \log\left|\left(t-\frac{3}{2}\right)+\sqrt{t^2-3t+2}\right|+C$$ **1**

$$= \log\left|\left(\sin\theta-\frac{3}{2}\right)+\sqrt{\sin^2\theta-3\sin\theta+2}\right|+C$$ ½

30. Solve the following Linear Programming Problem graphically :

Minimise $z = 3x + 8y$

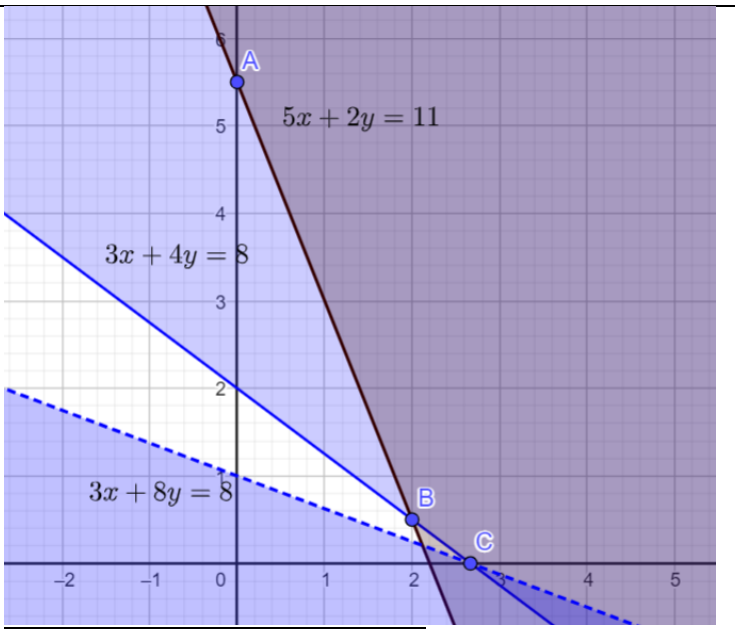
subject to the constraints

$$3x + 4y \geq 8$$

$$5x + 2y \geq 11$$

$$x \geq 0, y \geq 0$$

Sol.



Correct graph 1 mark

Corner Point	$z = 3x + 8y$
$A\left(0, \frac{11}{2}\right)$	**44**
$B\left(2, \frac{1}{2}\right)$	**10**
$C\left(\frac{8}{3}, 0\right)$	**8**

1½

since $3x + 8y < 8$ do not have any point in common with the feasible region,

$z_{min} = 8$ when $x = \frac{8}{3}, y = 0$

½

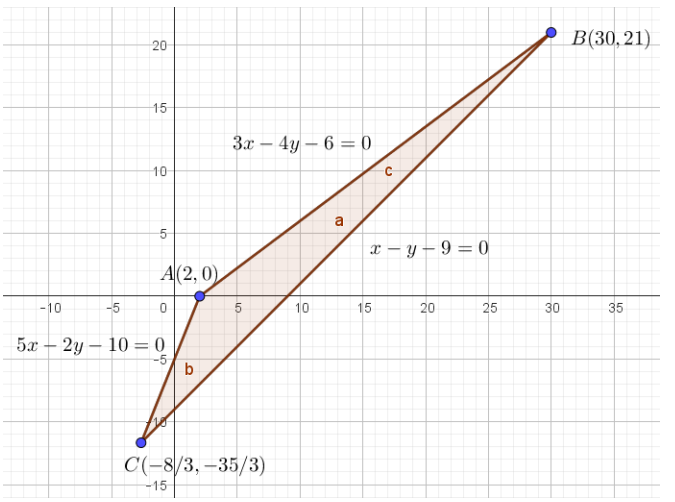
31. Find :

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)}\, dx$$

Sol.	$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$ Let $\frac{2x^2+1}{x^2\left(x^2+4\right)}=\frac{2y+1}{y\left(y+4\right)}$, where $x^2=y$ Put $\frac{2y+1}{y\left(y+4\right)}=\frac{A}{y}+\frac{B}{y+4}$ $\Rightarrow 2y+1=A\left(y+4\right)+By$ $\Rightarrow A=\frac{1}{4}, B=\frac{7}{4}$ $\therefore \frac{2y+1}{y\left(y+4\right)}=\frac{1}{4y}+\frac{7}{4\left(y+4\right)}=\frac{1}{4x^2}+\frac{7}{4\left(x^2+4\right)}$ $\Rightarrow I=\frac{1}{4}\int \frac{1}{x^2}dx+\frac{7}{4}\int \frac{1}{x^2+4}dx$ $=-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+C$	½ **1** ½ ½ ½
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32.	If matrix $A=\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations : $3x + 2y + z = 2000$ $4x + y + 3z = 2500$ $x + y + z = 900$	

Sol.	$\|A\| = 3(-2) - 4(1) + 1(5) = -5 \neq 0 \Rightarrow A^{-1}$ exists.	**1**
	$A_{11} = -2, A_{12} = -1, A_{13} = 3$ $A_{21} = -1, A_{22} = 2, A_{23} = -1$ $A_{31} = 5, A_{32} = -5, A_{33} = -5$ $adjA = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$	**1 ½**
	$A^{-1} = \frac{1}{\|A\|} adjA = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$	**1**
	Given system of equations can be written as $AX = B$, where $B = \begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix}$	
	$X = A^{-1}B$	½
	$= -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -2000 \\ -1500 \\ -1000 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \\ 200 \end{bmatrix}$	½
	$\therefore x = 400, y = 300$ and $z = 200$	½
33(a).	Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.	

Sol.	$\text{line}1: \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \quad ...(1)$ $\text{line}\,2: \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \quad ...(2)$ General points on (1) and (2) are $(3\lambda-1, 5\lambda-3, 7\lambda-5)$ and $(\mu+2, 3\mu+4, 5\mu+6)$ for the lines to intersect, $3\lambda - 1 = \mu + 2 \quad ...(3)$ $5\lambda - 3 = 3\mu + 4 \quad ...(4)$ $7\lambda - 5 = 5\mu + 6 \quad ...(5)$ solving (3) and (4) gives $\lambda = \frac{1}{2}$ and $\mu = -\frac{3}{2}$ clearly these values of λ and μ satisfies (5) $\Rightarrow$ given lines intersect. Point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$	**1** **1** **1** **1**
	OR	
33(b).	Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2.$	

Sol.	Given lines are $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-0}{1}$ and $\frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$ In vector form, lines are $\vec{r}=\left(\hat{i}-\hat{j}\right)+\lambda\left(2\hat{i}+3\hat{j}+\hat{k}\right)=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\left(-\hat{i}+2\hat{j}+2\hat{k}\right)+\mu\left(5\hat{i}+\hat{j}\right)=\vec{a}_2+\lambda\vec{b}_2$ now, $\vec{a}_2-\vec{a}_1=-2\hat{i}+3\hat{j}+2\hat{k}$ $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&1\\5&1&0\end{vmatrix}=-\hat{i}+5\hat{j}-13\hat{k}$ $\left\|\vec{b}_1\times\vec{b}_2\right\|=\sqrt{195}$ S.D. $=\left\|\frac{\left(\vec{a}_2-\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)}{\left\|\vec{b}_1\times\vec{b}_2\right\|}\right\|$ $=\left\|\frac{2+15-26}{\sqrt{195}}\right\|=\frac{9}{\sqrt{195}}$	**1** **1** ½ **1** ½ **1**
34.	Find the area of the triangle ABC bounded by the lines represented by the equations 5x – 2y – 10 = 0, x – y – 9 = 0 and 3x – 4y – 6 = 0, using integration method.	
Sol.	 	**Correct figure 1 mark**

	solving the given equations, the vertices of triangle are $A(2,0), B(30,21)$ and $C\left(-\frac{8}{3},-\frac{35}{3}\right)$ $\text{ar}(\triangle ABC)=\frac{3}{4}\int_2^{30}(x-2)\,dx-\int_9^{30}(x-9)\,dx+\left\lvert\int_{-\frac{8}{3}}^{9}(x-9)\,dx\right\rvert-\left\lvert\frac{5}{2}\int_{-\frac{8}{3}}^{2}(x-2)\,dx\right\rvert$ $=\frac{3}{8}(x-2)^2\Big]_2^{30}-\frac{1}{2}(x-9)^2\Big]_9^{30}+\left\lvert\frac{1}{2}(x-9)^2\Big]_{-\frac{8}{3}}^{9}\right\rvert-\left\lvert\frac{5}{4}(x-2)^2\Big]_{-\frac{8}{3}}^{2}\right\rvert$ $=294-\frac{441}{2}+\frac{1225}{18}-\frac{245}{9}=\frac{343}{3}$	**3** **1**
35(a).	Show that the relation S in set $\mathbb{R}$ of real numbers defined by $S = \{(a, b) : a \le b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$ is neither reflexive, nor symmetric, nor transitive.	
Sol.	We have $S=\{(a, b: a \le b^3\}$ where $a, b \in R$. (i) Reflexive: we observe that, $\frac{1}{2} \le \left(\frac{1}{2}\right)^3$ is not true. $\therefore \left(\frac{1}{2},\frac{1}{2}\right) \notin S$. So, S is not reflexive. (ii) Symmetric: We observe that $1 \le 3^3$ but $3 \not\le 1^3$ i.e., (1, 3) $\in S$ but (3, 1) $\notin S$. So, S is not symmetric. (iii) Transitive: We observe that, $10 \le 3^3$ and $3 \le 2^3$ but $10 \not\le 2^3$. i.e., (10, 3) $\in S$ and (3, 2) $\in S$ but (10, 2) $\notin S$. So, S is not transitive. $\therefore$ S is neither reflexive nor symmetric, not transitive.	**1 ½** **1 ½** **2**
	OR	

35(b).	Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].	
Sol.	$R=\{(a,b):\text{both } a \text{ and } b \text{ are either odd or even}\}$ for reflexive : Let $a \in A$ clearly both a and a are either odd or even $\therefore (a,a) \in R \Rightarrow R$ is reflexive. for symmetric : Let $a,b \in A$. Let $(a,b) \in R$ $\Rightarrow$ both a and b are either odd or even $\Rightarrow$ both b and a are either odd or even so, $(a,b) \in R \Rightarrow (b,a) \in R \Rightarrow R$ is symmetric. for transitive : Let $a,b,c \in A$. Let $(a,b) \in R, (b,c) \in R$ $\Rightarrow$ both a and b are either odd or even & both b and c are either odd or even $\Rightarrow$ both a and c are either odd or even so, $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R \Rightarrow R$ is transitive. equivalence class of $[1]=\{1,3,5,7\}$	**1** **1** **2** **1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions :

(i) Find the probability that the age of the selected student is a composite number. *1*

(ii) Let X be the age of the selected student. What can be the value of X ? *1*

(iii) (a) Find the probability distribution of random variable X and hence find the mean age. *2*

OR

(iii) (b) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number. *2*

Sol.

$(i)\ P(\text{age of selected student is a composite number})$

$= P(\text{age is } 14, 15 \text{ or } 16) = \frac{6}{10} = \frac{3}{5}$ **1**

$(ii)\ X$ can be $14, 15, 16, 17, 19$ **1**

$(iii)(a)$

X	**14**	**15**	**16**	**17**	**19**
$P(X)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$

1

$\text{mean} = \sum X.P(X)$

$= 14\left(\frac{1}{10}\right)+15\left(\frac{2}{10}\right)+16\left(\frac{3}{10}\right)+17\left(\frac{2}{10}\right)+19\left(\frac{2}{10}\right) = 16.4 \text{ years}$ **1**

OR

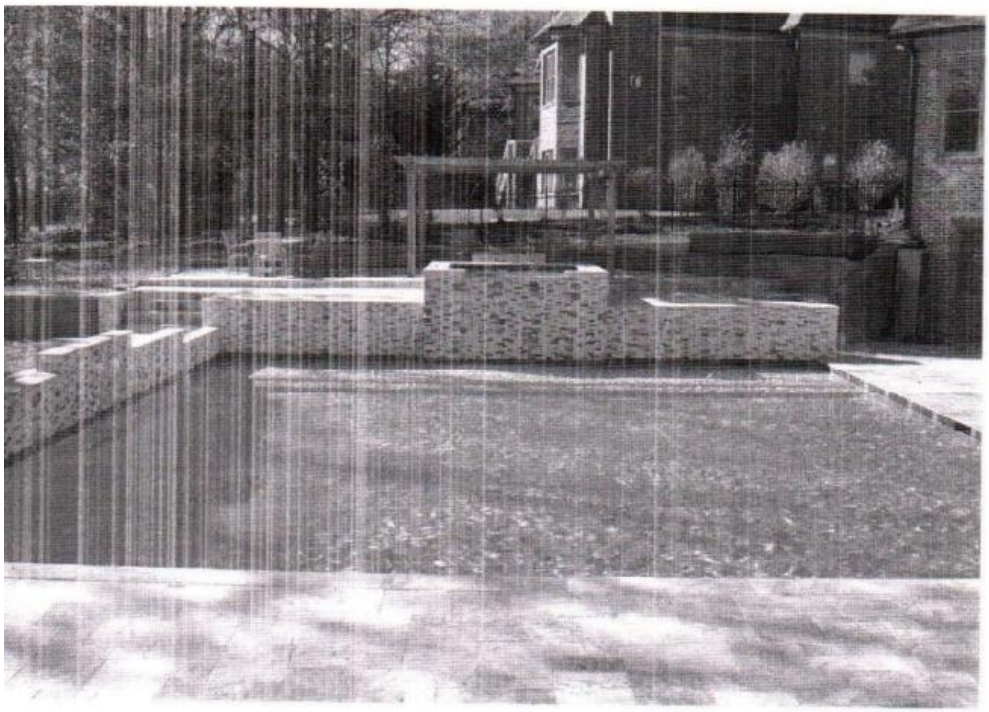
$(iii)(b)$ A : getting Prime number $= \{17,19\}$

B : age is greater than 15 years $= \{16,17,19\}$ **1**

$A \cap B = \{17,19\}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{3}$ **1**

37. A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is ₹ 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is ₹ 4000 $(\text{depth})^2$.



Suppose the side of the square plot is x metres and depth is h metres.
On the basis of the above information, answer the following questions :

(i) Write cost C(h) as a function in terms of h. *1*

(ii) Find critical point. *1*

(iii) (a) Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ? *2*

OR

(iii) (b) Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost. *2*

Sol.

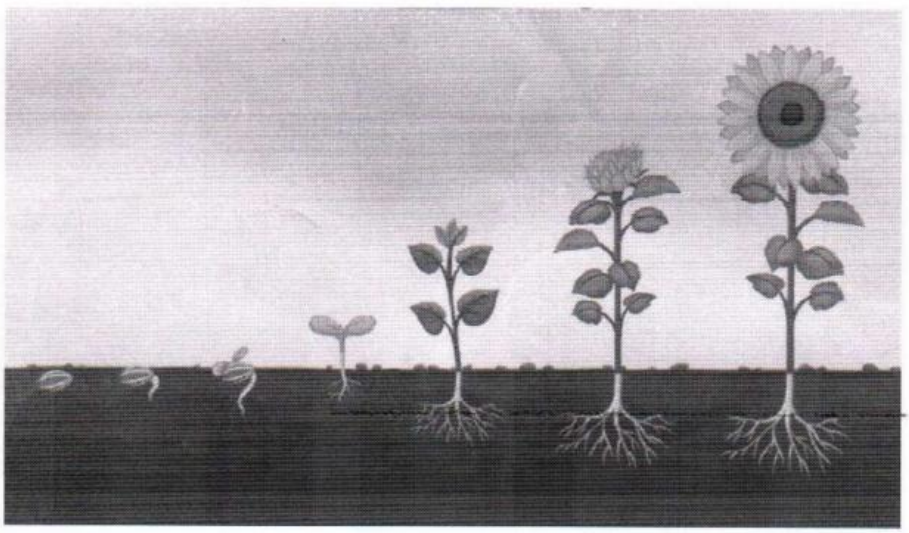
	(i) Capacity = area×depth $= x^2h = 250 \Rightarrow x^2 = \frac{250}{h}$ C (cost) $= 500x^2 + 4000h^2$ $\Rightarrow C = 500\left(\frac{250}{h}\right) + 4000h^2 = \frac{125000}{h} + 4000h^2$	**1**
	$(ii)\ \frac{dC}{dh} = -\frac{125000}{h^2} + 8000h$	
	$\frac{dC}{dh} = 0 \Rightarrow h = \frac{5}{2}m$ or $2.5\,m$	**1**
	$(iii)(a)\ \frac{d^2C}{dh^2} = -125000\left(\frac{-2}{h^3}\right) + 8000 = \frac{250000}{h^3} + 8000$	
	$\left.\frac{d^2C}{dh^2}\right\|_{h=2.5m} > 0 \Rightarrow$ Cost is minimum when $h = 2.5\,m$	**1**
	Minimum cost $= C = \frac{125000}{\left(\frac{5}{2}\right)} + 4000\left(\frac{5}{2}\right)^2 = Rs.\,75,000$	**1**
	OR	
	$(iii)(b)$ we already have found above that $h = \frac{5}{2}m$ when $\frac{dC}{dh} = 0$	
	for the values of h less than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} < 0$	**1**
	and, for the values of h more than $\frac{5}{2}$ and close to $\frac{5}{2}$, $\frac{dC}{dh} > 0$	
	By first derivative test, there is a minimum at $h = \frac{5}{2}$	
	Now, $x^2 = \frac{250}{h} \Rightarrow x^2 = \frac{250}{\left(\frac{5}{2}\right)} = 100 \Rightarrow x = 10\,m$	½
	also, $x = 4h$	½

38. In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2,\ 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions :

(i) What are the critical points of the function f(x) ? 2

(ii) Using second derivative test, find the minimum value of the function. 2

Sol. (i) $f'(x) = x^2 - 8x + 15 = (x-3)(x-5)$ **1**

$f'(x) = 0 \Rightarrow x = 3, 5$ are the critical points. **1**

(ii) Now $f''(x) = 2x - 8$

$f''(3) < 0$ and $f''(5) > 0$

so, mimimum value of $f(x)$ is at $x = 5$. **1**

$\text{min. value} = f(5) = \frac{5^3}{3} - 4(5)^2 + 15(5) + 2 = \frac{56}{3}$ **1**